

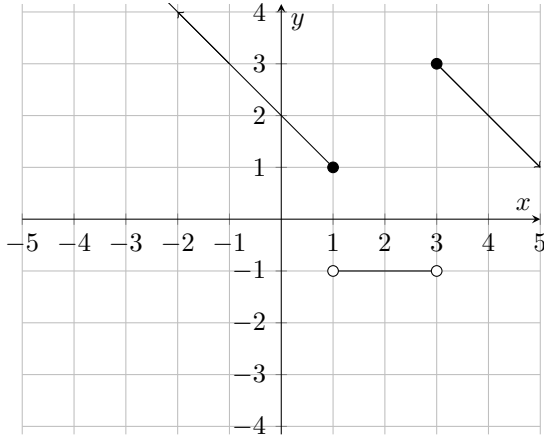
**Math-109: Pre-Calculus Algebra**  
**Section: 8**  
**Final Exam**

Name: \_\_\_\_\_

ULID: \_\_\_\_\_

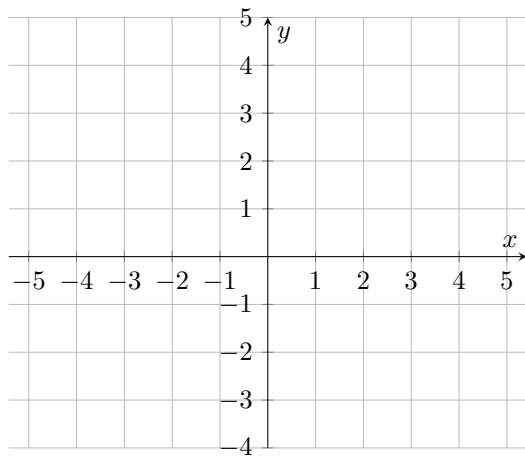
Solve only 20 problems. Clearly circle the problems that you want me to grade. Write complete step by step solutions to the problems. Partial credit will be given for the work shown.

1. Consider the following graph. Note that there are arrows at the end.



- (a) Is this a one-to-one function?
- (b) Determine the domain of the function. (Note the arrows at the end).
- (c) Determine the range of the function. (Note the arrows at the end).
- (d) Determine the intervals where the function is increasing, decreasing and constant.
- (e) Find all  $x$  in the domain such that  $f(x) = 2$ .
2. Determine whether the following functions are even, odd or neither. Show work
- (a)  $f(x) = 5x^2 + 2|x| + 3$
- (b)  $g(x) = \frac{1}{x} + x$
- (c)  $f(x) = x^2 - x$
3. Let  $f(x) = \frac{1}{2x+1}$  and  $g(x) = x^2 + 3$ . Find the expressions for the following and simplify:
- (a)  $f + g$
- (b)  $f - g$
- (c)  $\frac{f}{g}$
- (d)  $g(f(1))$
4. Evaluate the following for the given functions  $f(x) = x^3 - 1$ ,  $g(x) = |x| + 1$ .
- $\frac{f(2)-g(1)}{g(-1)}$
5. Find the difference quotient for the function  $f(x) = x^2 - 5x + 4$ .

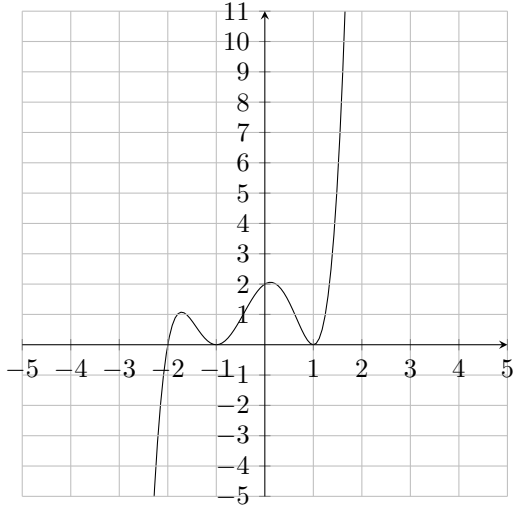
6. Write the expression of the function whose graph is transformed accordingly.
- The graph of  $y = 2^x$  reflected about the  $y$ -axis, and then shifted left 4 units.
  - The graph of  $x^2$  horizontally stretched by a factor of 3, shifted down by 2 units, and then reflected about the  $x$ -axis.
7. Find the inverse of  $f(x) = \frac{1}{x-10}$ .
8. Let  $f(x) = x^2 + 2x + 3$ .
- Find the vertex.
  - Write the quadratic function in standard(vertex) form.
  - Find the coordinates of the x-intercepts(zeros).
  - Find the coordinates of the y-intercept.
  - Sketch the graph and label all the intercepts and vertex.



9. Find the quadratic function that has the given vertex and goes through the given point. Write your answer in vertex form.  
Vertex:  $(1, -4)$ , Point:  $(1, 3)$ .
10. Are the following functions polynomials? If they are polynomials, determine their degrees.
- $f(x) = x^5(x - 3)^{10}(x + 9)$
  - $f(x) = x^2 - \frac{1}{x} + 11$
  - $f(x) = x^{10} - x^5 + x^2 + x^{2000} + 9$
  - $g(x) = \frac{3(x-3)^2(x-4)(x-9)}{x^2}$
11. Find a polynomial of minimum degree that has the given zeros.
- 1, 2
  - $-12, \frac{9}{4}, -\frac{1}{5}$

- (c) -10 (multiplicity 3), 7 (multiplicity 9)
- (d) 3 (multiplicity 5), 8 (multiplicity 1), 0 (multiplicity 2)

12. Consider the given graph of a polynomial. Imagine there are arrows at the two ends of the graph



- (a) List each real zero and its smallest possible multiplicity.
  - (b) Determine whether the degree of the polynomial is even or odd.
  - (c) Is the leading coefficient positive or negative?
  - (d) Find the y-intercept.
13. Divide the following. You can use synthetic division for (b).
- (a)  $x^3 - 12x^2 - 42 \div x^2 - 2x + 1$
  - (b)  $x^4 + 9x^3 - 4x + 9 \div x - 1$ .
14. Let  $P(x) = x^5 + 2x^4 - x - 2$ .
- (a) Use Descartes' rule of signs to determine the possible number of positive zeros for  $P(x)$ .
  - (b) Use Descartes' rule of signs to determine the possible number of negative zeros for  $P(x)$ . (Note that you have to find  $P(-x)$ .)
  - (c) Use the rational zero test to determine the possible rational zeros.
  - (d) Factor the polynomial as a product of linear and/or irreducible quadratic factors. (Note that you don't need to test for each and every possible rational zero.)

15. Find a polynomial of minimum degree that has the given zeros. Get rid of the imaginary  $i$ 's in the expression.

Zeros: 1, 0, 4,  $1 - i$ ,  $1 + i$ ,  $i$ ,  $-i$ .

16. Solve the following quadratic equations. Find all solutions including complex zeros.

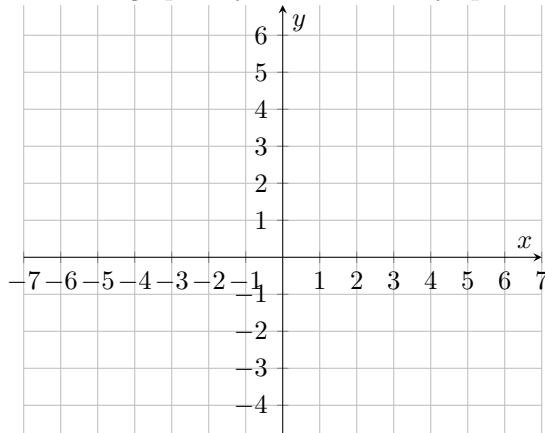
- (a)  $x^2 + x - 6 = 0$
- (b)  $(x + 5)^2 = -9$
- (c)  $(x + 25)^2 = 16$
- (d)  $5x^2 - 2x + 4 = 0$

17. Find the domain of the following rational functions.

- (a)  $f(x) = \frac{1}{x-1}$
- (b)  $f(x) = \frac{2x+18}{4x^2-16}$
- (c)  $f(x) = \frac{x^3+x^2+1}{x^2-10x-16}$
- (d)  $f(x) = \frac{x^5+x^4+x^3+x^2+x+1}{x^2+25}$

18. For the rational function  $f(x) = \frac{5x+1}{x-2}$ , find:

- (a) Find any  $x$ -intercepts.
- (b) Find any  $y$ -intercepts.
- (c) Find the equation of the vertical asymptote.
- (d) Find the equation of the horizontal asymptote.
- (e) Sketch the graph of  $f$  and label all asymptotes and intercepts.



19. Solve the given system of linear equations.

$$\begin{aligned} -4x + 3y &= 23 \\ 12x + 5y &= 1 \end{aligned}$$

$$\begin{aligned} 15x + 9y &= 3 \\ 5x + 3y &= 4 \end{aligned}$$

$$\begin{aligned}\frac{1}{3}x - \frac{1}{12}y &= 2 \\ 4x - 3y &= 5\end{aligned}$$

20. Write the augmented matrix if given the system of linear equations, and the system of equations if given the augment matrix.

(a)

$$\begin{aligned}4x + 2y - 3z &= 1 \\ 4x + z &= -1 \\ x + y - 4z &= 7\end{aligned}$$

(b)

$$\left( \begin{array}{cccc|c} 0 & 1 & 1 & -1 & 7 \\ 2 & 0 & 3 & 0 & 1 \\ 9 & 1 & 2 & 4 & -6 \end{array} \right)$$

21. Solve the following system using Gauss-Jordan elimination with back substitution.

$$\begin{aligned}3x - y + 5z &= 9 \\ x - 2y + 4z &= 1 \\ 2x + y &= 6\end{aligned}$$

22. Find the exact values of  $x$  for the following equations:

- (a)  $\log_3(x) = -2$   
 (b)  $\log_4(256) = x$   
 (c)  $\log_x(5) = \frac{1}{2}$   
 (d)  $\log_{x^2+1}(4) = -1$   
 (e)  $\log_{x-1}(27) = 3$

23. Write each expression as a sum or difference of  $\log(x)$ ,  $\log(y)$  and  $\log(z)$ .

- (a)  $\log(x^2\sqrt{y}z^3)$   
 (b)  $\log\left(\frac{x^2}{y^{\frac{1}{3}}z^5}\right)$

24. Write each expression as a single logarithm.

- (a)  $2\log(x) + 3\log(y) - \frac{1}{2}\log(z)$   
 (b)  $\log(x) - 2\log(y^2) + 3\log(z^2) - \log(12)$

25. Solve each equation exactly. Decimal values are not needed.

- (a)  $3^{x-1} = 81$   
 (b)  $10^{2x-3} = 7$   
 (c)  $\left(\frac{1}{3}\right)^y = 27$   
 (d)  $\ln(x) + \ln(x-1) = \ln(30)$

26. How much money should you put into a savings account now that earns 8% interest compounded monthly, if you want to have \$500,000 in 25 years? Round to the nearest cent.

27. How many years will it take your money to quadruple if it is invested at 12% continuously?