

Math-109: Pre-Calculus Algebra
Section: 8
Midterm Exam 2

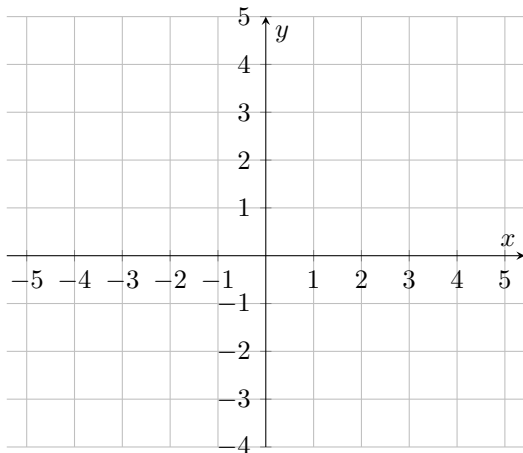
Name: _____

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Please write complete step by step solutions (whenever possible) to the problems below.

1. Let $f(x) = x^2 + 4x + 3$.

- (a) Find the vertex.
- (b) Write the quadratic function in standard(vertex) form.
- (c) Find the coordinates of the x-intercepts(zeros).
- (d) Find the coordinates of the y-intercept.
- (e) Sketch the graph and label all the intercepts and vertex.



2. Find the quadratic function that has the given vertex and goes through the given point. Write your answer in vertex form.

Vertex: $(3, -5)$, Point: $(5, 7)$.

3. Are the following functions polynomials? If they are polynomials, determine their degrees.

(a) $f(x) = x^7(x - 3)^{11}(x + 9)$

(b) $f(x) = x^2 - \frac{1}{x} + 11$

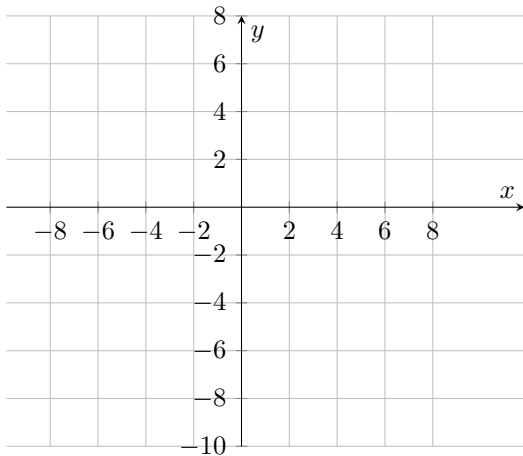
(c) $f(x) = x^{10} - x^5 + x^2 + \sqrt[3]{x} + 9$

(d) $g(x) = \frac{3(x-3)^2(x-4)(x-9)}{x}$

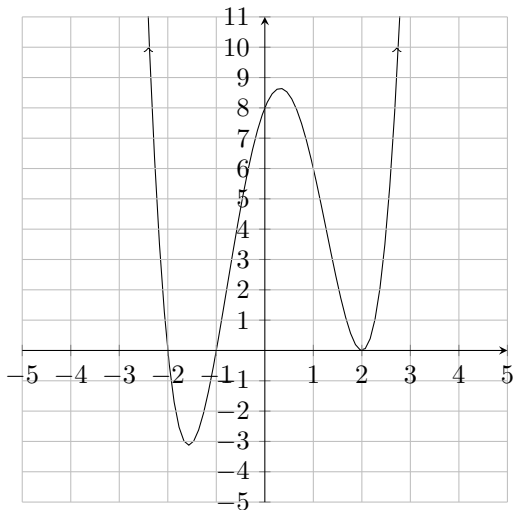
4. Consider the polynomial $P(x) = x(x + 2)(x - 5)^2(x - 11)^9$.

- (a) What is the degree of $P(x)$?

- (b) List all the zeros of $P(x)$ including their multiplicities.
5. Find a polynomial of minimum degree that has the given zeros.
- (a) $-1, \frac{3}{4}, -\frac{1}{6}$
- (b) -10 (multiplicity 3), 7 (multiplicity 9)
6. Let $f(x) = x^3 - x^2 - 6x$.
- (a) Find the zeros of the polynomial with respective multiplicities.
- (b) Determine whether the graph touches or crosses at each zero(x- intercept).
- (c) Find the y- intercept.
- (d) Determine the end behaviors.
- (e) Sketch the graph.



7. Consider the given graph of a polynomial.



- (a) List each real zero and its smallest possible multiplicity.
 - (b) Determine whether the degree of the polynomial is even or odd.
 - (c) Is the leading coefficient positive or negative?
 - (d) Find the y-intercept.
 - (e) Write the equation for the polynomial. (Assume the least degree possible.)
8. Divide the following. You can use synthetic division for (b).
- (a) $x^3 + 6x^2 - 2x - 5 \div x^2 - 1$
 - (b) $x^4 - x^3 - 2x + 2 \div x + 1$.
9. Let $P(x) = x^5 - 2x^4 + x^3 - 2x^2 - 2x + 4$.
- (a) Use Descartes' rule of signs to determine the possible number of positive zeros for $P(x)$.
 - (b) Use Descartes' rule of signs to determine the possible number of negative zeros for $P(x)$. (Note that you have to find $P(-x)$.)
 - (c) Use the rational zero test to determine the possible rational zeros.
 - (d) Factor the polynomial as a product of linear and/or irreducible quadratic factors. (Note that you don't need to test for each and every possible rational zero.)

BONUS PROBLEMS

1. A spoon of wine is poured from a barrel of wine into a (not full) glass of tea. After that, the same spoon of the (inhomogeneous) mixture from the glass is taken back into the barrel. Now both in the barrel and in the glass there is a certain volume of the foreign liquid (wine in the glass and tea in the barrel). In which is the volume of the foreign liquid greater: in the glass or in the barrel?
2. A downstairs panel contains three on-off switches, one of which controls the lamp in the attic- but which one? Your mission is to do something with the switches, then determine after only one trip to the attic which switch is connected to the attic lamp.