Name: _____

ULID: _____

Please write complete step by step solutions to the problems below.

1. Consider the following graph:
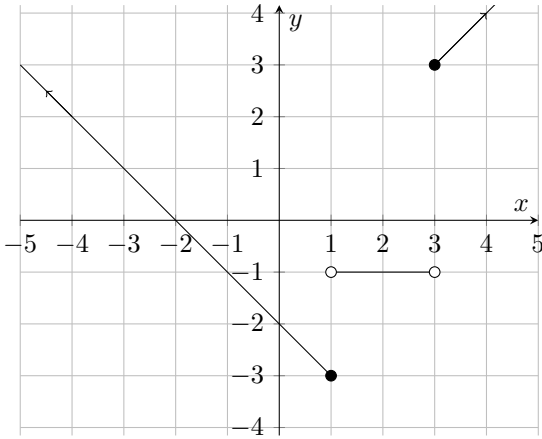
   (a) Is the graph a function?

   (b) Is this a one-to-one function?

   (c) Determine the domain of the function. (Note the arrows at the end).

   (d) Determine the range of the function. (Note the arrows at the end).

   (e) Determine the intervals where the function is increasing, decreasing and constant.

   (f) Find all x in the domain such that $f(x) = -1$.

2. Determine the domains of the following functions:

   (a) $f(x) = \frac{x-1}{x-9}$

   (b) $g(x) = \frac{1}{\sqrt{x+2}}$

3. Determine whether the following functions are even, odd or neither. Show work

   (a) $f(x) = 2x^2 + |x| + 1$

   (b) $g(x) = \frac{1}{x} + 3x$

4. Let $f(x) = 3x + 1$ and $g(x) = x^2 + 3$. Find the expressions for the following and simplify:
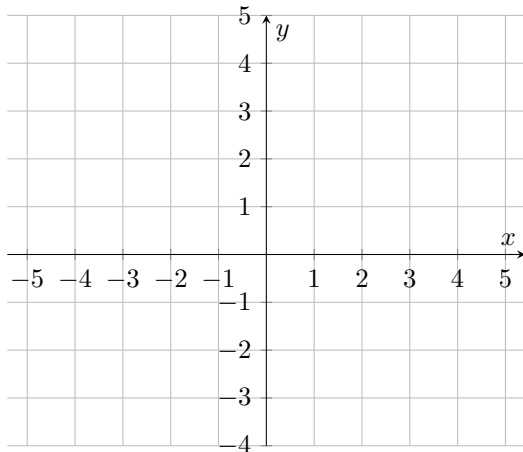
   (a) $f + g$

   (b) $f - g$

   (c) $\frac{f}{g}$

   (d) $g(f(1))$

5. Evaluate the following for the given functions $f(x) = x^3 - 1$, $g(x) = |x| + 1$.
   $\frac{f(2) - g(-1)}{g(1)}$
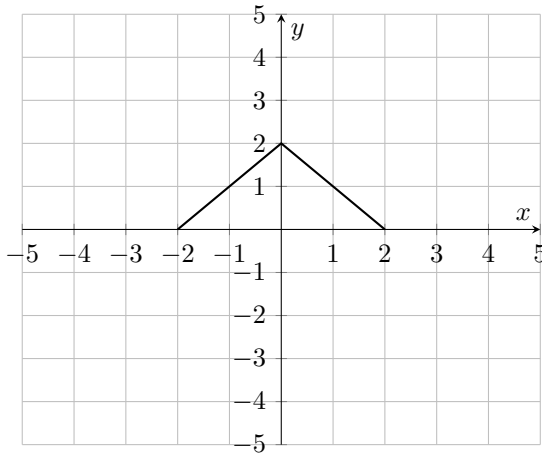
6. Graph the piecewise-defined function.

$$f(x) = \begin{cases} x^2, & x < -1, \\ 2x + 1, & -1 \le x < 1, \\ x^3, & 1 \le x. \end{cases}$$

7. Find the difference quotient for the function $f(x) = x^2 + 2x$.

8. Refer to the graph of $y = f(x)$ in the accompanying figure to sketch the graph of $y = f(-x - 2) + 1$

9. Write the expression of the function whose graph is transformed accordingly.

   (a) The graph of $y = \sqrt{x}$ reflected about the x-axis, and then shifted left 4 units.
   (b) The graph of x^3 horizontally stretched by a factor of $\frac{1}{2}$, shifted down by 2 units, and then reflected about the x-axis.

10. Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{1-x}$, find

   (a) $(f \circ g)(x)$
   (b) $(g \circ f)(x)$

11. Find the inverse of $f(x) = \frac{1}{3x+5}$.

12. Determine whether the function $f(x) = 2(\frac{x}{2}+1)^2 - 1$ is one to one. You can either show it algebraically or graphically. If you choose to do it graphically, then you must show the appropriate step by step transformations of the common function clearly.

BONUS QUESTIONS

1. Use transformations of functions to plot the graph of $f(x) = 2 - \frac{1}{3(1-\frac{x}{2})}$. Write the step by step transformations.

2. You are given 80 coins of the same denomination; you know that one of them is counterfeit and that it is lighter than the others. Locate the counterfeit coin by using four weighings on a pan balance.