Final Exam

Instructions: Write your solutions on **paper**, scan it and upload it to canvas as a single **pdf** file. Show complete work (show all the steps) and make sure that your scan is legible. Label your solutions and make sure they are in increasing order.

Remark. *Don't write answers in decimals. Find exact answers using fractions or radicals.*

1. [20 points] Find the general solutions of

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^3 y^2$$

2. [20 points] Find the general solutions of the following differential equation:

$$y'' + y' - 30y = 0$$

3. [20 points] Write the following system of equations in matrix form:

   (a)

$$\begin{aligned} x' &= 3x + y \\ y' &= 2y - x \end{aligned}$$

   (b)

$$\begin{aligned} x' &= 5tx + y - t^2 z + t^3 \\ y' &= ty - t^2 z + 1 \\ z' &= x + e^t y - z + \sin(t) \end{aligned}$$

4. [10 points] Solve

$$y' + \cos(x)y = \cos(x) \text{ for } y(0) = 1$$

5. [10 points] For the following:

   - Find the eigenvalues and eigenvectors.
   - Draw the vector field.
   - Plot at least two solutions (Not required to solve).

$$x_1' = 5x_1 + 2x_2$$
$$x_2' = 2x_1 - 3x_2$$

6. [10 points] Find the inverse Laplace transform of $\frac{3s+4}{s^2-s-6}$.

7. [10 points] Solve the equation

$$x''(t) - x(t) = t^3, \quad x(0) = -1, \quad x'(0) = 0$$

8. [10 points] Solve the equation

$$x''(t) - x(t) = (t+1)u(t-2), \quad x(0) = 0, \quad x'(0) = 0$$

   Here $u(t)$ is the Heaviside function.