

Midterm Exam 3

Time: 75 minutes

Instructions: Show complete work. Label your solutions and make sure they are in increasing order.

Remark. *Don't write answers in decimals. Find exact answers using fractions or radicals.*

1. [20 points] Write the following system of equations in matrix form:

(a)

$$\begin{aligned}x' &= 2tx + y \\y' &= 2y - x\end{aligned}$$

(b)

$$\begin{aligned}x' &= 2tx + y - t^2z \\y' &= y - t^2z + 1 \\z' &= x + e^tz - e^t\end{aligned}$$

2. [20 points] Write the following equation/system as a system of **first order** differential equations:

(a) $y''' - y' + y = \sin(t)$

(b)

$$\begin{aligned}x'' - 3y' + 2x &= \cos(t)e^t \\2y'' + 10x' - y &= e^t\end{aligned}$$

3. [10 points] Solve the following system.

$$\begin{aligned}x_1' &= 8x_1 + 3x_2 \\x_2' &= 3x_1 - 4x_2\end{aligned}$$

4. [10 points] Solve the following system.

$$\begin{aligned}x_1' &= 2x_1 - 3x_2 \\x_2' &= 3x_1 + 2x_2\end{aligned}$$

5. [10 points] For each of the following:

- Find the eigenvalues and eigenvectors
- Draw the vector field
- Plot at least two solutions (Not required to solve)

(a)

$$\begin{aligned}x_1' &= 3x_1 + x_2 \\x_2' &= x_1 - x_2\end{aligned}$$

(b)

$$\begin{aligned}x_1' &= -5x_1 + x_2 \\x_2' &= 4x_1 - 2x_2\end{aligned}$$

6. [10 points] Are the following vectors linearly dependent or independent? Justify your answer with proof.

$$\begin{bmatrix} 1 \\ t \\ 5t \end{bmatrix}, \begin{bmatrix} -t \\ te^{2t} \\ t^2 \end{bmatrix}, \begin{bmatrix} t \\ te^t \\ t^3 \end{bmatrix}$$