Midterm Exam 2

Time: 75 minutes

Instructions: Show complete work. Label your solutions.

Remark. *Don't write answers in decimals. Find exact answers using fractions or radicals.*

1. [20 points] Find the general solutions of the following differential equations:

$$y'' - 10y' - 11y = 0$$

2. [20 points] Find the general solutions of the following differential equations:

$$y'' + 4y' + 4y = 0$$

3. [10 points] Find the solution of the following differential equation:

$$2y'' - 6y' + 5y = 0 \text{ for } y(0) = 1 \text{ and } y'(0) = 1$$

4. [10 points] Find the general solutions of

$$y''' - 2y'' + 2y' - 4y = 0$$

5. [10 points] Consider a mass and spring system with a mass $m = 1$, spring constant $k = 2$, and damping constant $c = 3$.

(a) Set up and find the general solution of the system.

(b) You stretched the mass so that the initial position is $x(0) = 5$ and released it with a velocity $x'(0) = 0$ (You just released it from rest). Find the solution.

(c) Either by **sketching the graph of the solution** or **by doing the calculus+algebra**, explain what happens to the mass in the limit $t \to \infty$. Choose one of the three options for the explanation:

- It oscillates infinitely often and will never settle at the equilibrium.
- It oscillates infinitely often but will eventually tend towards the equilibrium.
- It doesn't oscillate around the equilibrium and will eventually tend towards at the equilibrium.

6. [10 points] Find the general solutions of

$$y'' + 2y' - 8y = sin(2t)$$