

Midterm Exam 1

Time: 75 minutes

Instructions: Show complete work whenever possible.

1. [20 points] Verify that $x(t) = te^{-5t}$ is a solution to the following differential equation:

$$x''(t) + 10x'(t) + x(t) = -24te^{-5t}$$

2. [20 points] For each of the four equations, answer the following questions:

- (a) Identify the dependent variable and the independent variable.
- (b) What is the order of the equation?
- (c) Is it linear or non-linear?
- (d) If it is linear, is it homogeneous or non-homogeneous? If it is non-linear write N/A for this question
- (e) Is it an equation with constant coefficients or non-constant coefficients?

- i. $y''' + 5x^2y'' + 10y + 10x = 0$
- ii. $\frac{d^4y}{dt^4} - \frac{d^3y}{dt^3} - e^ty = \frac{d^2y}{dt^2}$
- iii. $\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}y + 7y = 0$
- iv. $x'' - 5x' + e^{10}x = 0$

3. [10 points] Solve

$$x' = e^{\cos(t)} \sin(t) \text{ for } x(0) = \pi$$

4. [10 points] Find the general solutions of

$$\frac{dy}{dx} = x^3y$$

5. [10 points] Solve

$$y' + 6x^5y = e^{-x^5} \text{ for } y(1) = 1$$

6. [12 points] For the differential equation $y' = (y + 1)(y - 1)$,
- (a) Draw the phase diagram.
 - (b) Find the critical points and classify them as stable or unstable.
 - (c) Draw the slope field and sketch the solution with $y(0) = -2$
 - (d) Find $\lim_{t \rightarrow \infty} y(t)$ for the solution with initial condition $y(0) = -2$
7. [10 points] Find the general solutions of

$$y' + \frac{1}{x}y = y^2$$