Homework 2

Instructions: Read the homework instructions on the syllabus. Write your solutions on paper or a writing tablet, scan it and upload it to canvas. The file must be in pdf extension. Show neat and complete work and make sure that your scan is legible. Label your solutions and make sure they are in increasing order.

1. Solve

$$xy' = y + 2x^2y$$
 for $y(1) = 5$ (1)

2. Solve

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x\cos(t) \text{ for } x(0) = \pi \tag{2}$$

3. Solve

$$y' + 3x^2y = \sin(x)e^{-x^3}$$
 for $y(0) = 1$ (3)

4. Find the general solutions of

$$y' + \sin(x)y = \tan(x) \tag{4}$$

5. Solve

$$y' + 3x^2y = \sin(x)e^{-x^3}$$
 for $y(0) = 1$ (5)

- 6. Consider $x' = x^2$ (Warning! x is the dependent variable. So you can take t as the independent variable. Draw x vs. t)
 - (a) Draw the phase diagram (this is not the slope field).
 - (b) Find the critical points and mark them stable or unstable.
 - (c) Sketch typical solutions of the equation using the slope field. (Sketch a solution for each region of the phase space)
 - (d) Find $\lim_{x\to\infty} x(t)$ for the solution with initial condition x(0) = -1
- 7. Consider x' = sin(x). (Warning! x is the dependent variable. So you can take t as the independent variable. Draw x vs. t)

- (a) Draw the phase diagram for $-4\pi \le x \le 4\pi$.
- (b) Find the critical points and mark them **stable or unstable**.
- (c) Sketch typical solutions of the equation using the slope field. (Sketch a solution for each region of the phase space)
- (d) Find $\lim_{x\to\infty} x(t)$ for the solution with initial condition x(0) = -1

8. Consider
$$y' = (y-1)(y-2)y^2$$

- (a) Draw the phase diagram.
- (b) Find the critical points and mark them **stable or unstable**.
- (c) Sketch typical solutions of the equation using the slope field. (Sketch a solution for each region of the phase space)
- (d) Find $\lim_{x\to\infty} x(t)$ for the solution with initial condition x(0) = 1.5
- 9. Solve

$$y' + xy = y^4$$
 for $y(0) = -1$ (6)

Suggestion: First make sure you understand why this is neither linear nor separable.

10. Solve

$$y' + y(x^2 - 1) + xy^6 = 0 \text{ for } y(1) = 1$$
(7)

Suggestion: First make sure you understand why this is neither linear nor separable.