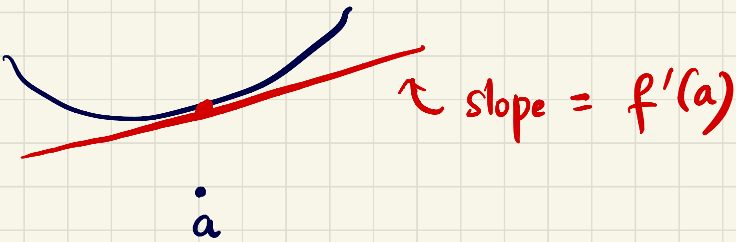
- Grades are posted
- Bonus assigment:
  Same questions as in Midterm 2
  worth 5% of Midterm Grade
  Due Next Tuesday March 16
  Wiley Plus (Copy of Midterm 2 (Bonus))
  Not Timed, unlimited attempts
- Tutoring
  2options 1) Student Learing Center
  (Lee Hall)
  2) Mathlab No appointment
  (Lee Hall)
  Links are in Moodle.
- Office Hours TTR 10:45 - 12:30
  (Link on Moodle)
- HW due next Tuesday.

3.1. Derivative Formulas for Powers and Polynomials

We know 2 ways of finding derivatives:

1. Slope of the tangent line.



2. Avg. rate of change where Δx is very small:

$$\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Δx is very small number

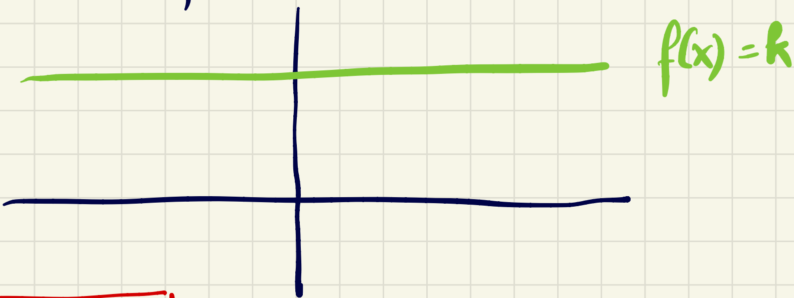
0.01 etc . .

These methods only give an estimate for the derivative.

Goal: To find exact values and formulas for derivatives of functions.

Derivative of Constant Function

$$f(x) = k$$

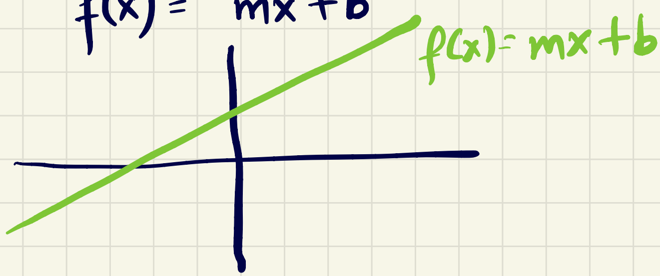


$\boxed{f'(x) = 0}$ because slope of horizontal line is 0 everywhere.

Ex: $\dfrac{d}{dx}(5) = 0$ $\left(\Leftrightarrow \begin{array}{l} f'(x) = 0 \text{ when} \\ f(x) = 5 \end{array} \right)$

Derivative of Linear Function

$$f(x) = mx + b$$



$f'(x) = m$ because the slope is m everywhere.

Ex: $\dfrac{d}{dx}\left(5 - \dfrac{3}{2}x\right) = -\dfrac{3}{2}$ $\left(\Leftrightarrow \begin{array}{l} f'(x) = -\dfrac{3}{2} \text{ when} \\ f(x) = 5 - \dfrac{3}{2}x \end{array} \right)$

Derivative of a Constant Times a function

$$\boxed{\begin{aligned} \frac{d}{dx}[c\,f(x)] &= c\,\frac{d}{dx}(f(x)) \\ &= c\,f'(x) \end{aligned}}$$

Pull out the constant c

c is a constant

Derivative of Sum and Difference

Let f and g are two functions.

$$\boxed{\begin{aligned} \frac{d}{dx}[f(x)+g(x)] &= \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x)) \\ &= f'(x) + g'(x) \end{aligned}}$$

Derivative of sum is the sum of the derivatives.

$$\boxed{\begin{aligned} \frac{d}{dx}[f(x)-g(x)] &= \frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x)) \\ &= f'(x) - g'(x) \end{aligned}}$$

Derivative of difference is the difference of the derivatives.

Powers of x

$$\boxed{\frac{d}{dx}(x^n) = n\,x^{n-1}}$$

n can be any number.

Ex: $f(x) = x^2$

$$f'(x) = \frac{d}{dx}(x^2)$$

$$= 2 \cdot x^{2-1} \qquad (n = 2)$$

$$= 2x^1$$

$$= \boxed{2x}$$

Ex $f(x) = x^3$

$$f'(x) = \frac{d}{dx}(x^3)$$

$$= 3\,x^{3-1} \qquad (\text{Power Rule};\ n=3)$$

$$= \boxed{3x^2}$$

Problem 1

Find the derivatives of

a) $f(x) = x^8$

Soln. $f'(x) = 8x^{8-1}$ (Power Rule)

$= \boxed{8x^7}$

b) $f(x) = \dfrac{1}{x^3}$

Soln. Note $\dfrac{1}{x^3} = x^{-3}$.

Thus, $f'(x) = \dfrac{d}{dx}(x^{-3})$

$= -3x^{-3-1}$ [Power rule; $n = -3$]

$= \boxed{-3x^{-4}}$

c) $f(x) = \sqrt{x}$

Soln. Note $\sqrt{x} = x^{\frac{1}{2}}$.

Thus, $f'(x) = \frac{1}{2} x^{\frac{1}{2} - 1}$ (Power Rule; $n = \frac{1}{2}$)

$$= \boxed{\frac{1}{2} x^{-1/2}}$$

$\downarrow$

Further simplification:

$$\frac{1}{2} \cdot \frac{1}{x^{1/2}}$$

$$= \frac{1}{2\sqrt{x}}$$

d) $f(t) = 2t^{4.5}$

<u>Soln.</u> $f'(t) = \frac{d}{dt}(2t^{4.5})$

$= 2\frac{d}{dt}(t^{4.5})$

$\left[\frac{d}{dx}[c f(x)] = c\frac{d}{dx}(f(x))\right]$; $c=2$

$= 2 \cdot 4.5\, t^{4.5-1}$

[Power Rule; $n = 4.5$]

$= \boxed{9t^{3.5}}$

e) $f(x) = 3x^5$

<u>Soln.</u> $f'(x) = \frac{d}{dx}(3x^5)$

$= 3\frac{d}{dx}(x^5)$

[Pulling out the constant coefficient]

$= 3 \cdot 5 \cdot x^{5-1}$

[Power Rule; $n=5$]

$= \boxed{15x^4}$

Derivatives of Polynomials

Polynomial is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

(Sum of power functions and constants)

Problem 2 Find derivatives of the following functions:

a) $f(p) = p^5 + p^3$

Soln. $f'(p) = \frac{d}{dp}(p^5 + p^3)$

$= \frac{d}{dp}(p^5) + \frac{d}{dp}(p^3)$ [Sum Formula]

$= 5p^{5-1} + 3p^{3-1}$ [Power Rules; $n = 5$ and $n = 3$]

$= \boxed{5p^4 + 3p^2}$

b) $f(x) = 5x^2 - 7x^3$

Soln. $f'(x) = \frac{d}{dx}(5x^2 - 7x^3)$

$= \frac{d}{dx}(5x^2) - \frac{d}{dx}(7x^3)$ [Sum Formula]

$= 5\frac{d}{dx}(x^2) - 7\frac{d}{dx}(x^3)$ [Pulling out constants]

$= 5 \cdot 2 \cdot x^{2-1} - 7 \cdot 3 \cdot x^{3-1}$ [Power Rule $n=2$ and $n=3$]

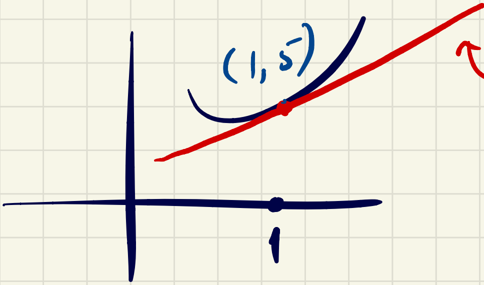
$= 10x^1 - 21x^2$

$= \boxed{10x - 21x^2}$

Problem 3 Find the equation for the tangent line at $x=1$ to the graph of

$$y = x^3 + 2x^2 - 5x + 7$$

Soln.



equation of line $= ?$

Equations of lines: 1. $y = mx + b$

vs. 2. $y - y_0 = m(x - x_0)$

When $x = 1$,

$$y = 1^3 + 2 \cdot 1^2 - 5 \cdot 1 + 7$$
$$= 1 + 2 - 5 + 7$$
$$= 5$$

Thus, $(x_0, y_0) = (1, 5)$

The slope $(m) = f'(1)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3+2x^2-5x+7\right)$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(2x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(7)$$

[Sum and Difference Formula]

$$= \frac{d}{dx}(x^3) + 2\frac{d}{dx}(x^2) - 5\frac{d}{dx}(x) + \frac{d}{dx}(7)$$

$$= 3x^{3-1} + 2.2x^{2-1} - 5.1.x^{1-1} + 0$$

$$= 3x^2 + 4x - 5.x^0 + 0$$

$$= 3x^2 + 4x - 5$$

$$f'(1) = m = 3\cdot 1^2 + 4\cdot 1 - 5$$

$$= 3+4-5 = 2$$

Equation of tangent:

$$y - 5 = 2(x - 1)$$

$$y - 5 = 2x - 2$$

$$y = 2x - 2 + 5$$

$$\boxed{y = 2x + 3}$$

3.2. Exponential and Logarithmic Function

1. $$\boxed{\frac{d}{dx}(a^x) = (\ln a)\, a^x}$$

2. $$\boxed{\frac{d}{dx} e^x = e^x}$$

This follows from 1.:

Take $a = e$, from 1 we get

$$\frac{d}{dx} e^x = (\ln e)\, e^x$$

$$= 1 \cdot e^x$$

$$= e^x$$

3. $$\boxed{\frac{d}{dt} e^{kt} = k\, e^{kt}}$$

k is a constant

4. $$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}}$$

Problem

Find derivative of

$$f(x) = 5 + 3x^2 - 7e^{-0.2x} + 2\ln x$$

Soln.

$$f'(x) = \frac{d}{dx}\left(5 + 3x^2 - 7e^{-0.2x} + 2\ln x\right)$$

$$= \frac{d}{dx}(5) + \frac{d}{dx}(3x^2) - \frac{d}{dx}\left(7e^{-0.2x}\right) + \frac{d}{dx}(2\ln x)$$

$$= \frac{d}{dx}(5) + 3\frac{d}{dx}(x^2) - 7\frac{d}{dx}\left(e^{-0.2x}\right) + 2\frac{d}{dx}(\ln x)$$

$$= 0 + 3\cdot 2\cdot x^{2-1} - 7(-0.2)\,e^{-0.2x} + 2\cdot\frac{1}{x}$$

$$= \boxed{6x + 1.4e^{-0.2x} + \frac{2}{x}}$$