HW 9 4.3 # 25abcd

$f(x) = 0.866x + \sin x$

Skip this question.

HW 8 3.3 # 8

$y = \sqrt{s^9+9}$

Find derivative.

Let $z = s^9 + 9$

$$\frac{dy}{ds} = \frac{d}{ds}\sqrt{s^9+9}$$

$$= \frac{d}{ds}\sqrt{z}$$

$$= \frac{d}{dz}\sqrt{z} \cdot \frac{dz}{ds}$$ (Chain Rule)

$$= \frac{1}{2} z^{-1/2} \cdot \frac{d(s^9+9)}{ds}$$

$$= \frac{1}{2} z^{-1/2} \cdot (9s^8) = \frac{1}{2\sqrt{s^9+9}} \cdot 9s^8$$

$$f(\theta) = (e^{\theta} + e^{-\theta})^{-2}$$

Let $z = e^{\theta} + e^{-\theta}$

$$\frac{df(\theta)}{d\theta} = \frac{d}{d\theta}(e^{\theta} + e^{-\theta})^{-2}$$

$$= \frac{d}{d\theta} z^{-2}$$

$$= \frac{d}{dz} z^{-2} \cdot \frac{dz}{d\theta} \qquad \text{(Chain Rule)}$$

$$= -2z^{-2-1} \cdot \frac{d(e^{\theta} + e^{-\theta})}{d\theta}$$

$$= -2z^{-3} \cdot \left(\frac{d}{d\theta}e^{\theta} + \frac{d}{d\theta}e^{-\theta}\right)$$

$$= -2z^{-3} \cdot \left(e^{\theta} + (-1)e^{-\theta}\right) \qquad \left[\text{Derivative of } e^{kx} = ke^{kx}\right]$$

$$= -2z^{-3}(e^{\theta} - e^{-\theta})$$

$$= -\frac{2}{z^3}(e^{\theta} - e^{-\theta})$$

$$= \boxed{\frac{-2}{(e^{\theta} + e^{-\theta})}(e^{\theta} - e^{-\theta})}$$

$$= \frac{-2}{e^{\theta}+e^{-\theta}} \cdot \left(e^{\theta}-e^{-\theta}\right)$$

Problem 1 Find the global maximum/minimum of the following functions.

a) $f(x) = x^3 - 9x^2 - 48x + 52$ on $-5 \leq x \leq 14$.

Soln. Find critical points:

$$f'(x) = \frac{d}{dx}\left(x^3 - 9x^2 - 48x + 52\right)$$

$$= \frac{d}{dx}(x^3) - \frac{d}{dx}9x^2 - \frac{d}{dx}48x + \frac{d}{dx}52$$

$$= 3x^2 - 9 \cdot 2x - 48 \cdot 1 + 0$$

$$= 3x^2 - 18x - 48$$

Solve $f'(x) = 0$

$$3x^2 - 18x - 48 = 0$$

2 ways:

1. Factor
2. Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4 \cdot 3(-48)}}{2 \cdot 3}$$

$$= \frac{18 \pm \sqrt{900}}{6}$$

$$= \frac{18 \pm 30}{6}$$

$$= \frac{18+30}{6}, \quad \frac{18-30}{6}$$

$$= 8, -2$$

The critical points are $8, -2$.

Find the values at critical points & endpoints:

$$f(8) = 8^3 - 9 \cdot 8^2 - 48 \cdot 8 + 52$$
$$= -396$$

$$f(-2) = (-2)^3 - 9(-2)^2 - 48(-2) + 52$$
$$= 104$$

$$f(-5) = (-5)^3 - 9(-5)^2 - 48(-5) + 52$$
$$= -58$$

$$f(14) = 14^3 - 9 \cdot 14^2 - 48 \cdot 14 + 52$$
$$= 360.$$

14 is the global maximum.

8 is the global minimum

b) $f(t) = te^{-t}$ for $t > 0$

Soln. Find critical points:

$$f'(t) = \frac{d}{dt}(te^{-t})$$

$$= \frac{d}{dt}t \cdot e^{-t} + t\frac{d}{dt}e^{-t} \quad \left(\text{Product Rule}\right)$$

$$= 1 \cdot e^{-t} + t \cdot (-1)e^{-t} \quad \left(\text{Derivative of } e^{Kt} = Ke^{Kt}\right)$$

$$= e^{-t} - te^{-t}$$

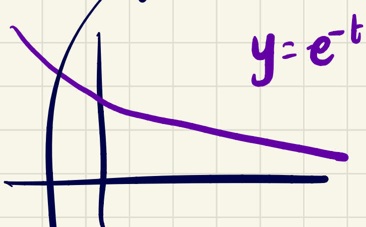
$$f'(t) = 0$$

$$e^{-t} - te^{-t} = 0$$

$$e^{-t}(1 - t) = 0$$

Either i) $e^{-t} = 0$ or ii) $1 - t = 0$

$t = 1$



No solution

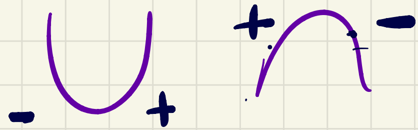
1 is the only critical point.

1 local min. or local max?

$f'(t) = e^{-t}(1-t)$

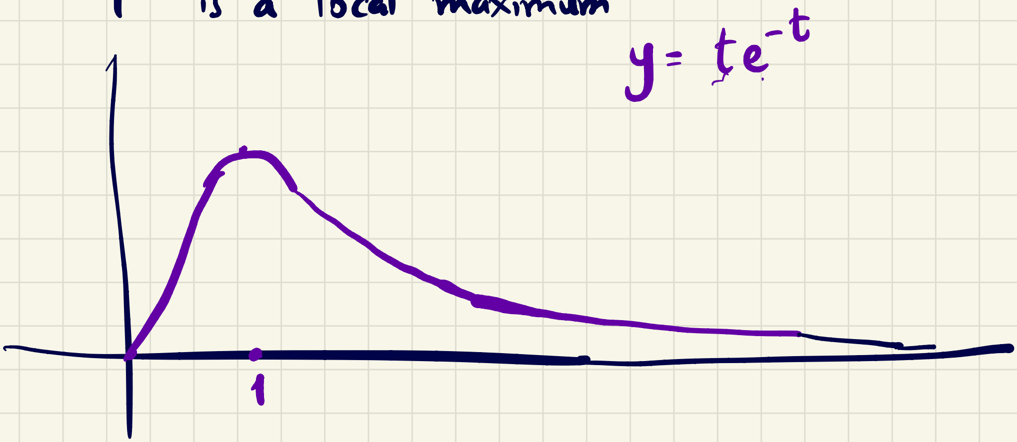
$t > 1, \quad f'(t) < 0$

$t < 1, \quad f'(t) > 0$



By first derivative test,

1 is a local maximum



$f(t)$ is decreasing for $t > 1$ (or $(1, \infty)$)
since $f'(t)$ is negative.

$f(t)$ is increasing for $0 < t < 1$ (or $(0, 1)$)
since $f'(t)$ is positive.

1 is the global maximum.
No global minimum.

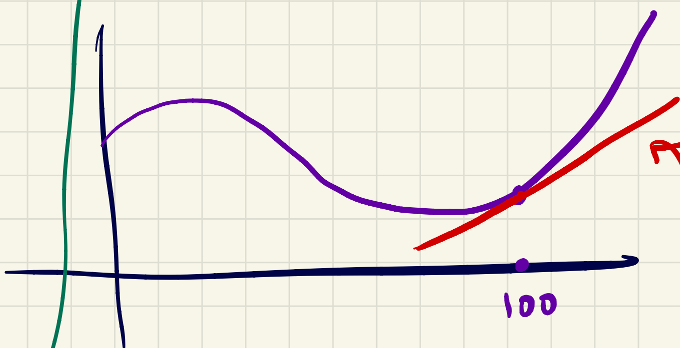
4.4. Profit, Cost, Revenue

Recall marginal cost (MC) = Derivative of cost function

$= C'(q)$

marginal revenue (MR) = Derivative of revenue function

$= R'(q)$



Slope of tangent $= MC(100)$

$\approx$ cost of adding 1 additional item.

$\approx$ revenue received when adding 1 additional item

Profit $\pi(q) = R(q) - C(q)$

Goal: Maximum and minimum profit.

Case 1

Endpoints included

Case 2

Endpoints not included

Problem The revenue from selling q items is $R(q) = 500q - q^2$ and the total cost is $C(q) = 150 + 10q$. Find the quantity which maximizes profit.

Soln. Profit $\pi(q) = R(q) - C(q)$

$$= 500q - q^2 - (150 + 10q)$$

$$= 500q - q^2 - 150 - 10q$$

$$= -q^2 + 490q - 150$$

Want: global maximum of $\pi(q)$
$(q \geq 0)$

Find critical points:

$$\pi'(q) = \frac{d}{dq}(-q^2 + 490q - 150)$$

$$= \frac{d}{dq}(-q^2) + \frac{d}{dq}490q - \frac{d}{dq}150$$

$$= -2q + 490$$

Solve $\pi'(q) = 0$

$$-2q + 490 = 0$$

$$\frac{2q}{2} = \frac{490}{2}$$

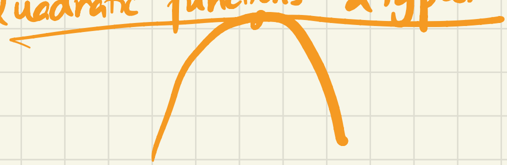
$$q = 245$$

$q = 245$ is the critical point.

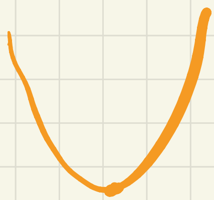
can use first derivative test / second derivative test to check whether 245 is local min/ local max.

$$\pi(q) = -q^2 + 490q - 150$$

Quadratic functions 2 types:



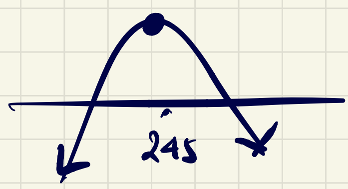
$a < 0$



$a > 0$

$ax^2 + bx + c$

$\pi(q)$ has a peak



$q = 245$ local maximum

Thus $\boxed{245}$ is a global maximum.