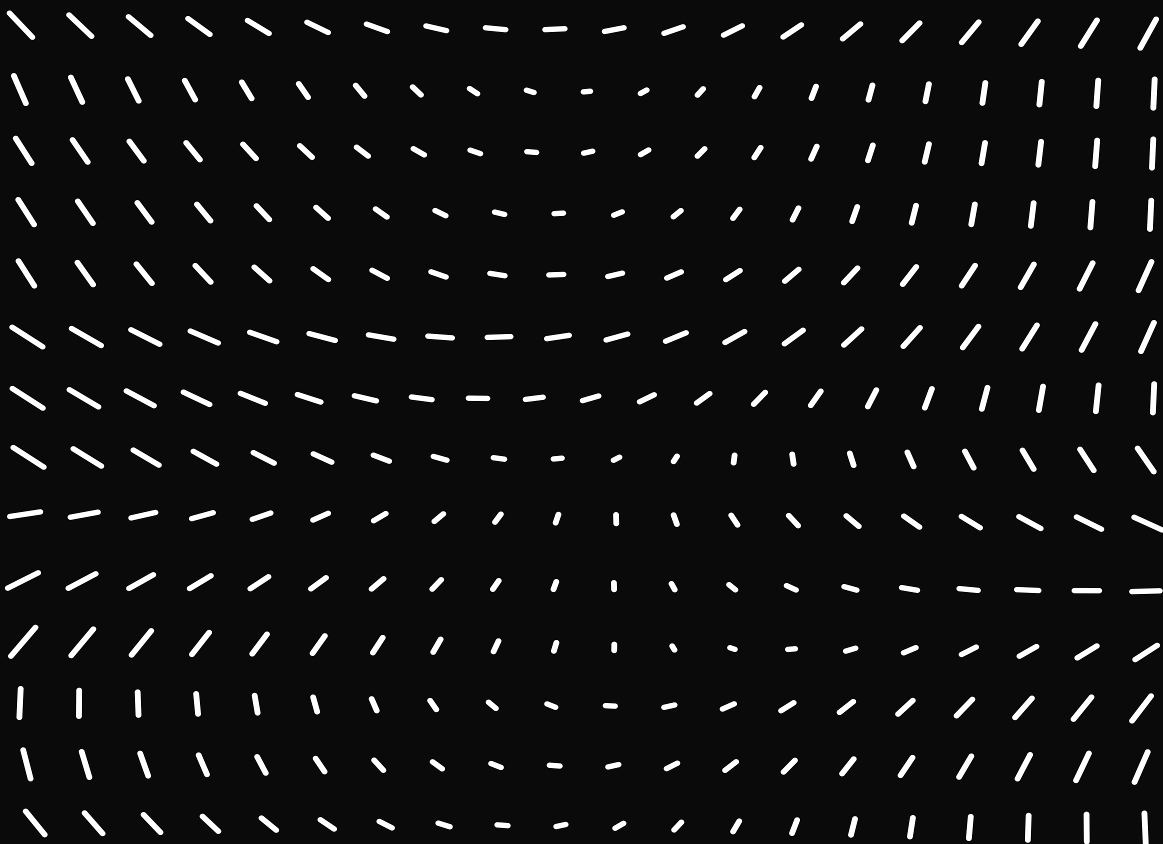


HW 8 Due Tonight ✓

Midterm 3 April 1, Thursday (Wiley Plus)

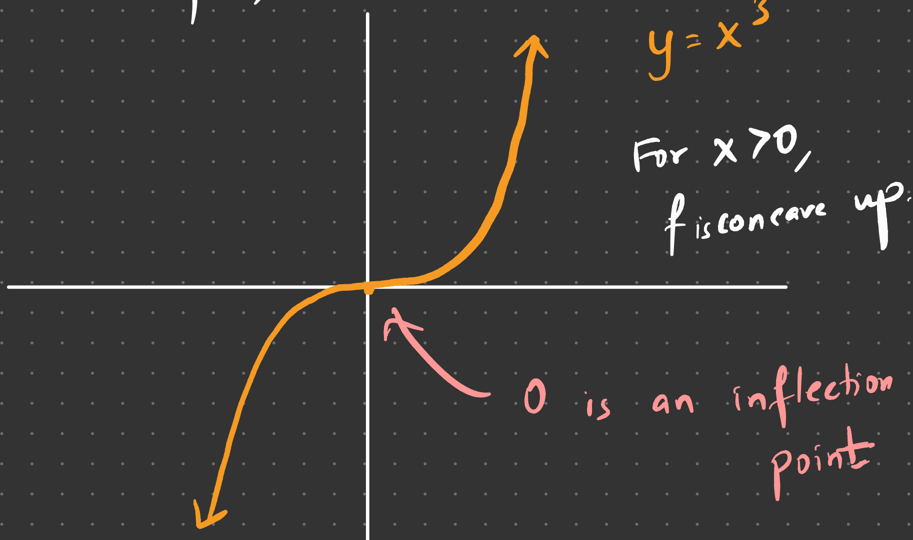
HW 9 Due Tuesday, March 30

HW 10 Due after exam ← cover the material for Midterm 3.

Sample Test Due April 1.

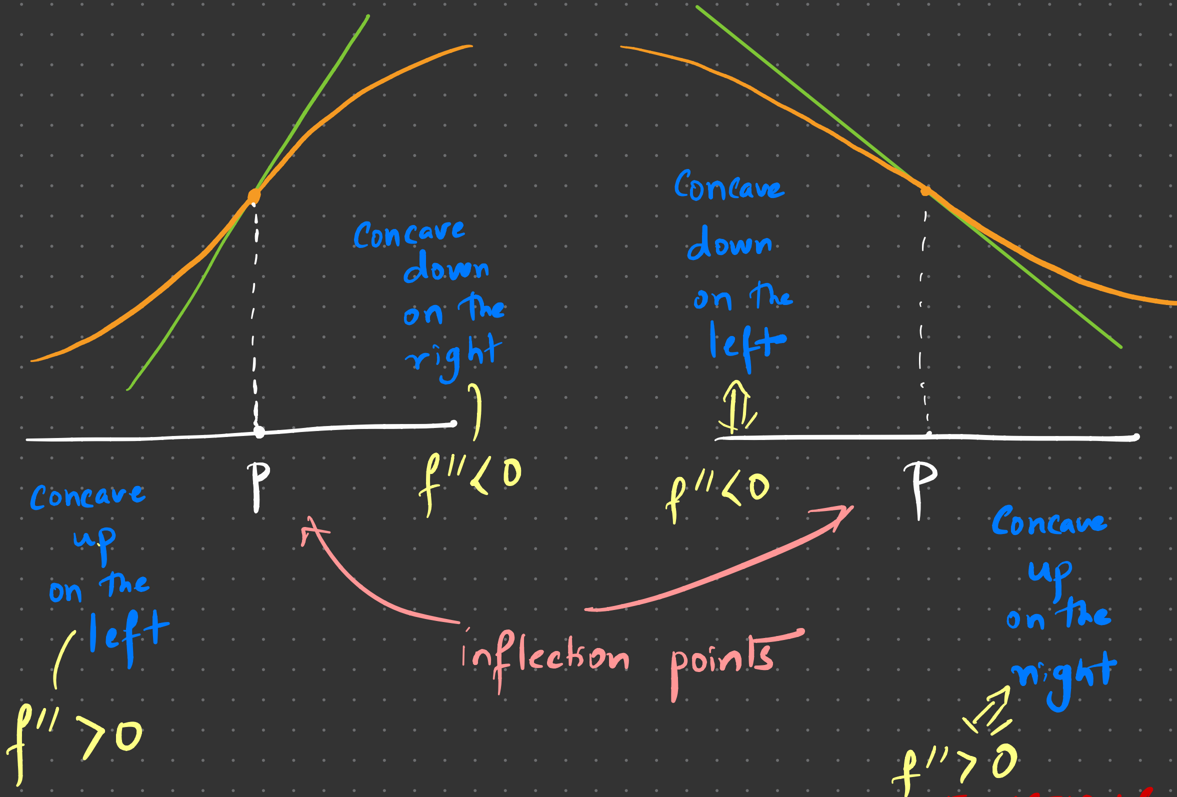
4.2. Inflection Points

$f(x) = x^3$



For $x < 0$,
f is concave down

Def. A point at which the graph of f changes concavity is called an inflection point of f



GOAL: TO FIND INFLECTION POINTS OF FUNCTIONS

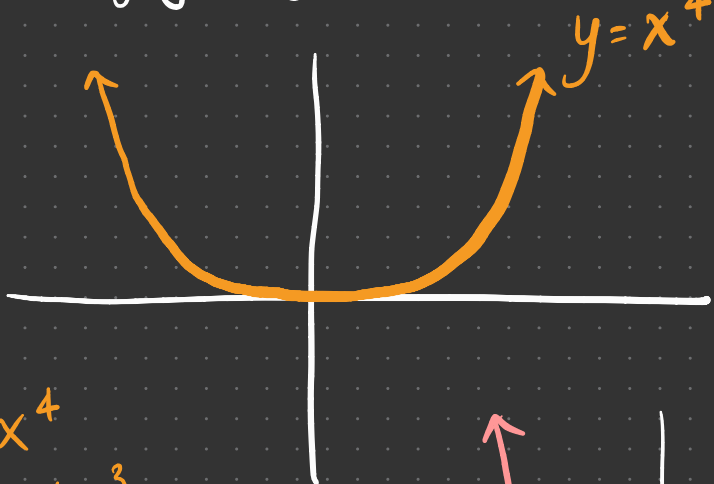
Since f'' is changing sign at p, $f''(p) = 0$

Thus, for an inflection point p, $f''(p) = 0$.

Warning:

$f''(x) = 0$ does not imply that x is an inflection point.

You would also have to check that the f'' is changing sign.

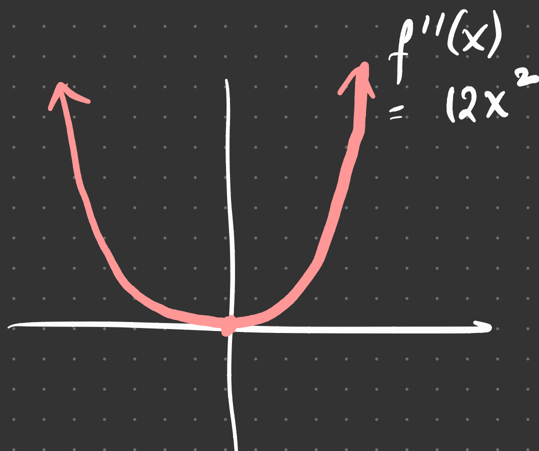


$f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''(0) = 12 \cdot 0^2 = 0$



We have $f''(0) = 0$ but 0 is not an inflection point because f'' does not change sign at 0.

Problem 1 Find the inflection points

a) $f(x) = x^3 - 9x^2 - 48x + 52$

Soln. Find points x such that $f''(x) = 0$:

$$f'(x) = \frac{d}{dx} f(x)$$

$$= \frac{d}{dx}(x^3) - \frac{d}{dx}(9x^2) - \frac{d}{dx}(48x) + \frac{d}{dx}(52)$$

$$= 3x^2 - 9 \cdot 2x - 48 + 0$$

$$= 3x^2 - 18x - 48$$

$$f''(x) = \frac{d}{dx} f'(x)$$

$$= \frac{d}{dx}(3x^2) - \frac{d}{dx}(18x) - \frac{d}{dx}(48)$$

$$= 3 \cdot 2x - 18 - 0$$

$$= 6x - 18$$

$$f''(x) = 0$$

$$6x - 18 = 0$$

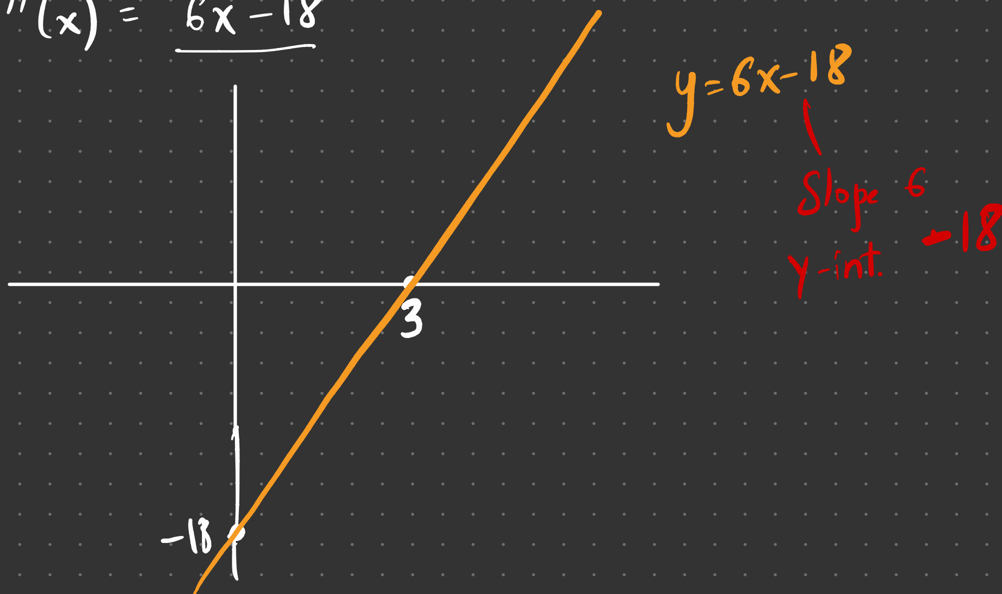
$$6x = 18$$

$$x = 3$$

3 is the only possible inflection point.

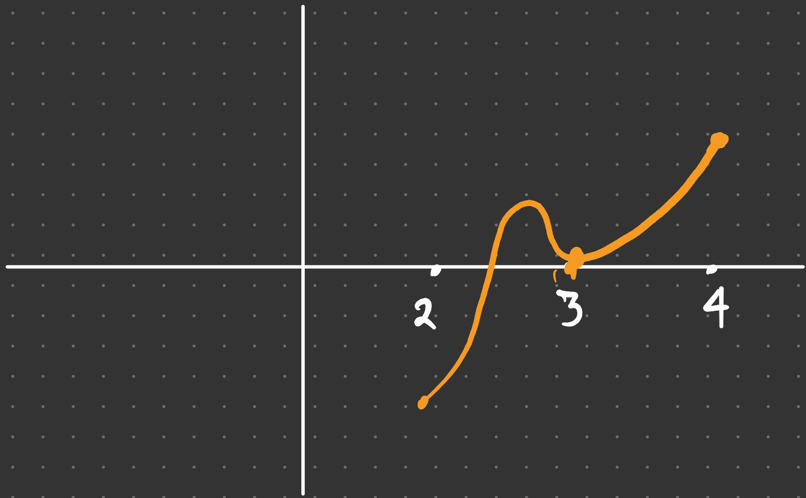
Check whether f'' changes sign at 3:

$f''(x) = \underline{6x - 18}$



Since f'' changes sign at 3, 3 is an inflection point.

$f''(x)$



2.99 3 3.01 4

b) $f(x) = x^5 - 5x^4 + 35$

Soln. $f'(x) = \frac{d}{dx}(x^5 - 5x^4 + 35)$

$= \frac{d}{dx}(x^5) - \frac{d}{dx}(5x^4) + \frac{d}{dx}(35)$

$= 5x^4 - 5 \cdot 4x^3 + 0$

$= 5x^4 - 20x^3$

$f''(x) = \frac{d}{dx}(5x^4 - 20x^3)$

$= \frac{d}{dx}(5x^4) - \frac{d}{dx}(20x^3)$

$= 5 \cdot 4x^3 - 20 \cdot 3x^2$

$= 20x^3 - 60x^2$

$f''(x) = 0$

$20x^3 - 60x^2 = 0$

$20x^2(x - 3) = 0$

Either: $20x^2 = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

0 and 3 are the possible inflection points.

0:

$$f''(x) = 20x^2(x-3)$$

$$f''(0.1) = 20(0.1)^2(0.1-3)$$
$$= \text{negative}$$

$$f''(-0.1) = 20(-0.1)^2(-0.1-3)$$
$$= \text{negative}$$

Since f'' is not changing sign at 0, 0 is not an inflection point.

3:

$$f''(x) = 20x^2(x-3)$$

On the right of 3, $f''(x) > 0$

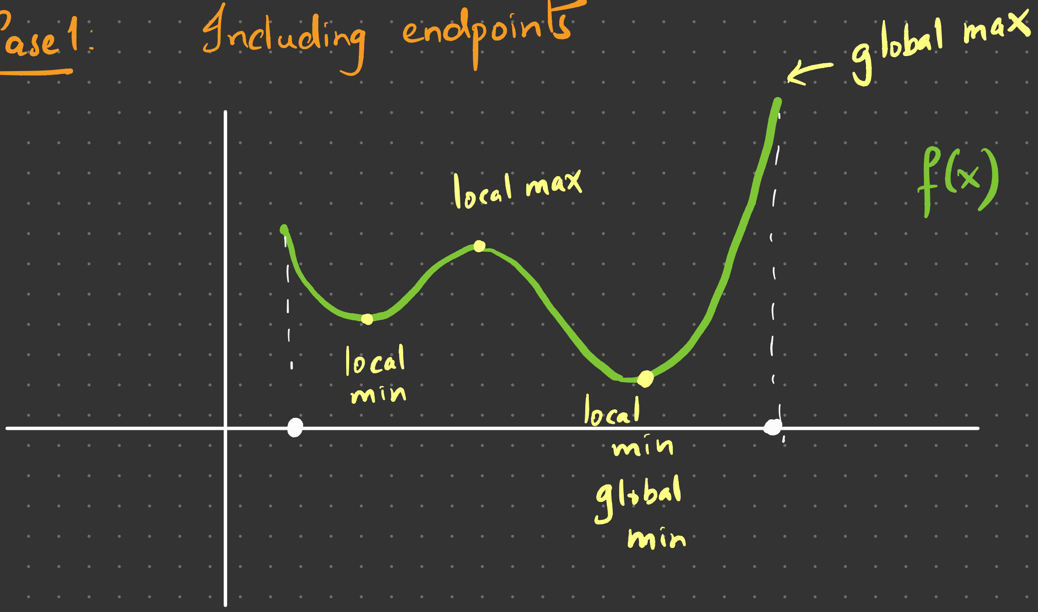
on the left of 3, $f''(x) < 0$.

Since f'' changes sign at 3, 3 is an inflection point.

4.3. Global Maxima and Minima

GOAL: TO FIND GLOBAL MINIMA AND MAXIMA

Case 1: Including endpoints

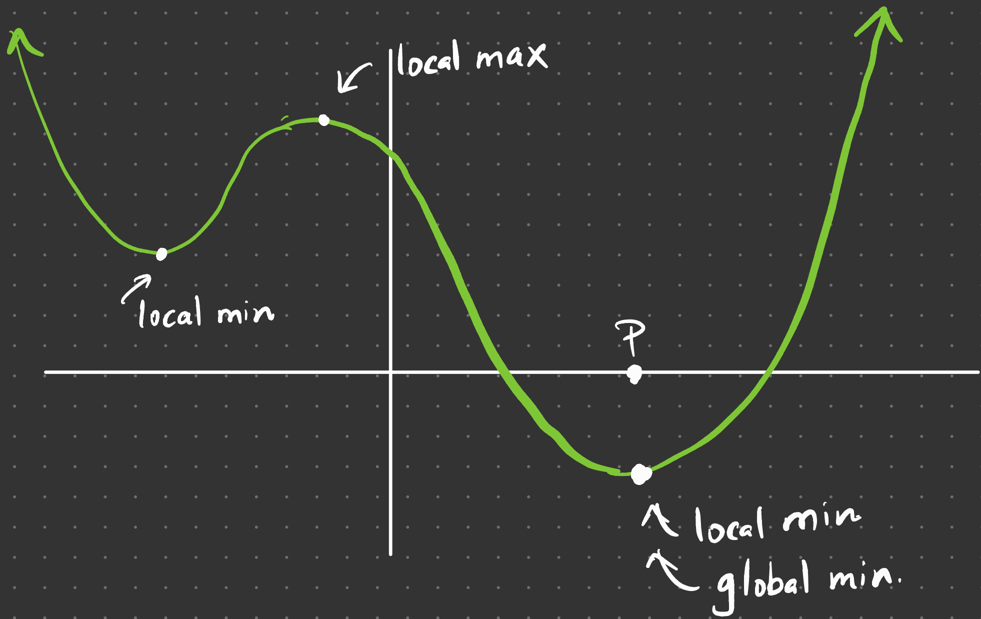


Strategy:
1. Find critical points
2. Compare values of f at the critical points and the endpoints.

global min / max $\overset{\text{implies}}{\Longrightarrow}$ local min / max

$\Longrightarrow$ critical point or end point

Case 2: Endpoints not included
Domain is the entire real line.



global minimum:

$x = p$

global maximum:

no global maximum.

Strategy
1. Find critical points.
2. Compare the values of critical points and check the behavior of f as $x \to \infty$ and $x \to -\infty$

Problem 1 Find global min/max of

$$f(x) = x^3 - 9x^2 - 48x + 52$$

on $-5 \leq x \leq 14$