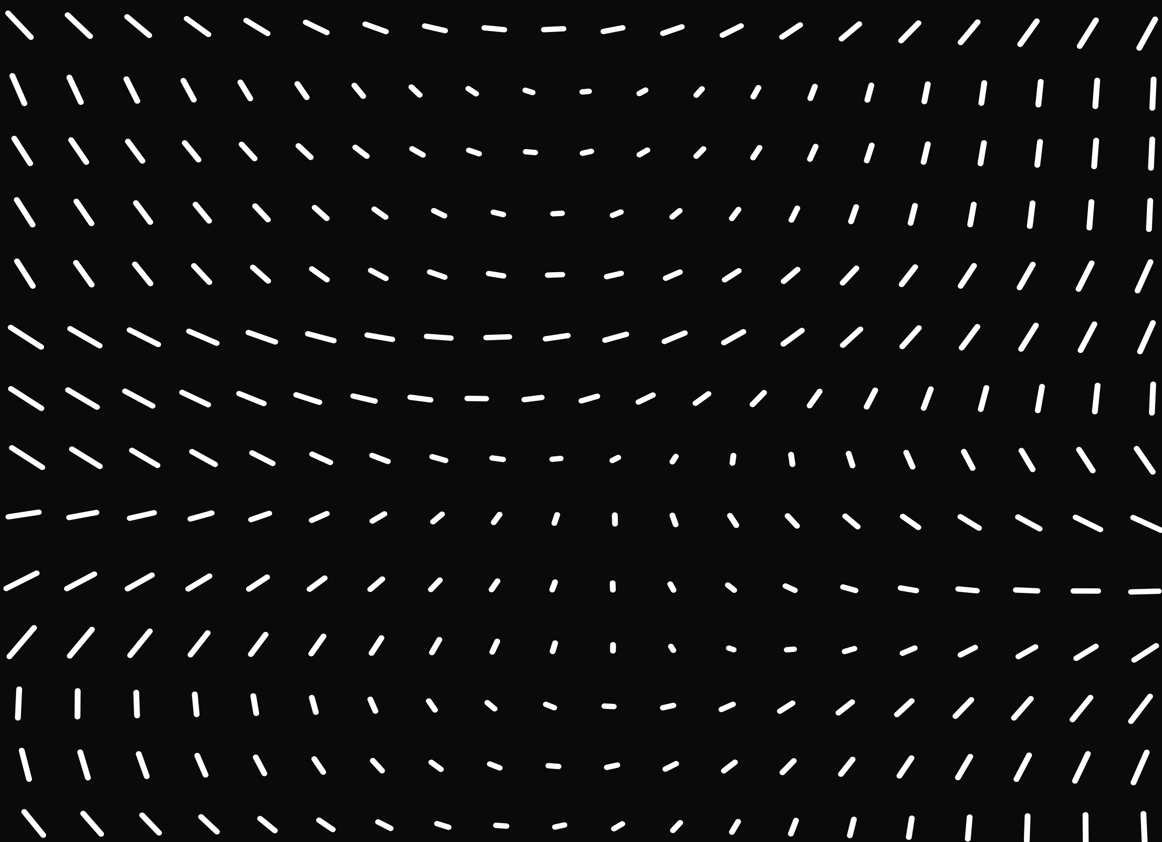


HW 7 Due Tonight

Bonus Assignment Due Tonight

3.4 The Product and Quotient Rules

$f(x)$ and $g(x)$ are functions.

We want to find the derivative of $f(x) \cdot g(x)$.

The Product Rule:

$$\boxed{(fg)' = f'g + fg'}$$

Memorize

Written differently,

$u = f(x)$, $v = g(x)$

$$\frac{d(uv)}{dx} = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$$

Same formula

Example 1

Differentiate

a) $x^2 e^{2x}$

Soln. Let $f(x) = x^2$ and $g(x) = e^{2x}$. Then

$$x^2 e^{2x} = f(x)\, g(x)$$

So $$\frac{d}{dx} x^2 e^{2x} = \frac{d}{dx}\left(f(x)\cdot g(x)\right)$$

$$(*) \quad = f'(x)\, g(x) + f(x)\, g'(x) \quad \text{(Product Rule)}$$

So, $(**)$ $f'(x) = 2x^{2-1}$ (Power Rule)

$$= 2x$$

$(***)$ $g'(x) = 2e^{2x}$ (Derivative of $e^{kx} = ke^{kx}$)

By $(*)$, $(**)$, $(***)$,

$$\frac{d}{dx} x^2 e^{2x} = 2x \cdot e^{2x} + x^2 \cdot 2e^{2x}$$

$$= \boxed{2x e^{2x} + 2x^2 e^{2x}}$$

b) $t^3 \ln(t+1)$

<u>Soln.</u> Let $f(t) = t^3$ and $g(t) = \ln(t+1)$.

Then $t^3 \ln(t+1) = f(t)\, g(t)$.

Hence, $\frac{d}{dt} t^3 \ln(t+1) = \frac{d}{dt}\left(f(t)\, g(t)\right)$

$= f'(t)\, g(t) + f(t)\, g'(t)$ $(*)$

(Product Rule)

Since $f(t) = t^3$,

$f'(t) = 3t^{3-1}$ (Power Rule)

$= 3t^2$

we know

$\left(\frac{d}{dx} \ln x = \frac{1}{x}\right)$

Now $g(t) = \ln(t+1)$

Let $z = t+1$

$\frac{d}{dt} g(t) = \frac{d}{dt} \ln(t+1)$

$= \frac{d \ln(z)}{dt}$

$= \frac{d}{dz} \ln(z) \cdot \frac{dz}{dt}$ (Chain Rule)

$$= \frac{1}{z} \cdot \frac{d(t+1)}{dt}$$

$$= \frac{1}{z} \cdot 1$$

$$= \frac{1}{t+1}$$

From (*) we have

$$\frac{d}{dt} t^3 \ln(t+1) = 3t^2 \cdot \ln(t+1) + t^3 \cdot \frac{1}{t+1}$$

$$= \boxed{3t^2 \ln(t+1) + \frac{t^3}{t+1}}$$

Quotient Rule

Let $f(x)$, $g(x)$ be two functions.

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

Memorize

Same thing

Written differently,

$$u = f(x) \quad , \quad v = g(x)$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

Example 2

Differentiate

a) $\dfrac{5x^2}{x^3+1}$

Soln. Let $f(x) = 5x^2$, $g(x) = x^3+1$. Then

$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) = \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

$$(*) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{g(x)^2} \quad \left(\text{Quotient Rule}\right)$$

Since $f(x) = 5x^2$,

$$f'(x) = 5 \cdot 2\,x^{2-1} \quad (\text{Power Rule})$$

$$= 10x$$

Since $g(x) = x^3+1$,

$$g'(x) = \frac{d}{dx}(x^3+1)$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(1)$$

$$= 3x^2 + 0 = 3x^2$$

By (*),

$$\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{g(x)^2} \quad \left(\text{Quotient Rule}\right)$$

$$= \frac{10x\,(x^3+1) - 5x^2 \cdot 3x^2}{(x^3+1)^2}$$

$$= \frac{10x^4 + 10x - 15x^4}{(x^3+1)^2}$$

$$= \boxed{\frac{-5x^4 + 10x}{(x^3+1)^2}}$$

b) $\dfrac{1}{1+e^x}$

Soln.

Ans. $\dfrac{e^x}{(1+e^x)^2}$

Ch 4

4.1 Local Maxima and Minima

Recall:

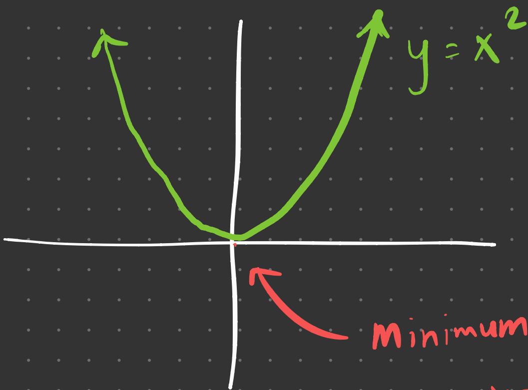
$f' > 0$ on an interval $\Rightarrow$ f is increasing on that interval.

$f' < 0$ " " " $\Rightarrow$ f " decreasing " " ".

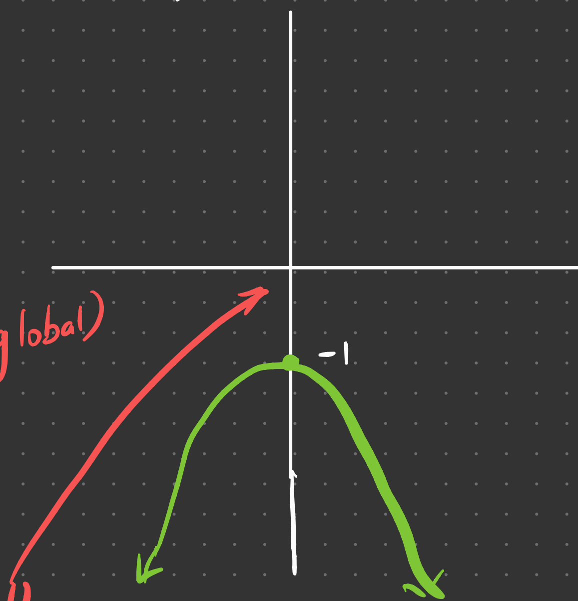
$f'' > 0$ " " " $\Rightarrow$ f is concave up on that interval

$f'' < 0$ " " " $\Rightarrow$ f is concave down on that interval.

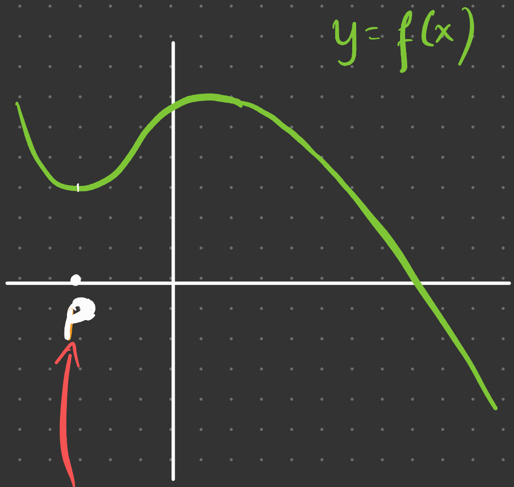
Consider $y = x^2$ $\qquad\qquad$ $y = -x^2 - 1$

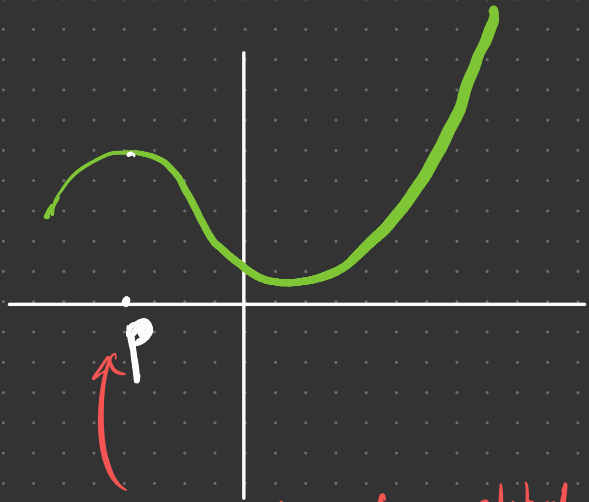


minimum (global) at $x = 0$



maximum (global) at $x = 0$





$x = p$ is not a global minimum as before because there are other points where the values of f are smaller.

But $x = p$ is a local minimum.

$x = p$ is not a global maximum. because there are other points where the values of f are greater.

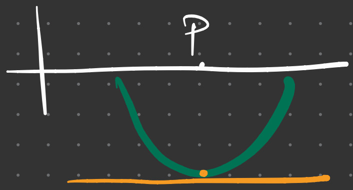
But $x = p$ is a local maximum

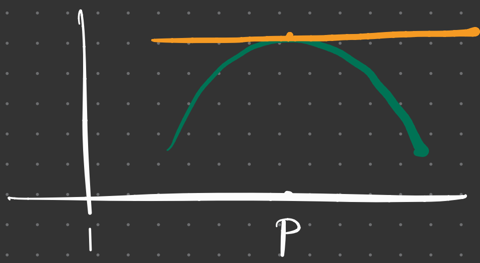
Def. f has <u>local minimum</u> at p if $f(p)$ is less than or equal to the value of f for points near p.

Def. f has <u>local maximum</u> at p if $f(p)$ is greater than or equal to the value of f for points near p.

Goal: Find the local minimas and local maximas of functions.

Notice that



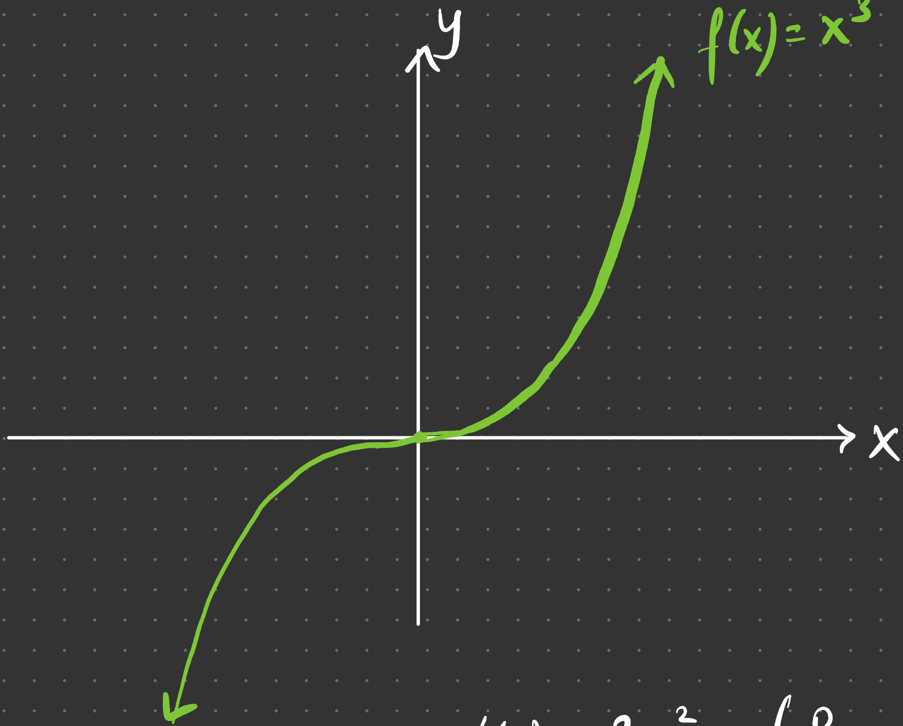


at local minimums and local maximums, the derivative $f'(p) = 0$.

But, is it true that if $f'(p) = 0$ at some point p, then p has to be a local minimum or maximum?

Ans. No. Not True

$f'(p) = 0 \not\Rightarrow$ p is a local minimum or local maximum.



$f'(0)$

$$f'(x) = 3x^2 \quad \text{(Power Rule)}$$

$$f'(0) = 3 \cdot 0^2 = 0$$

But clearly, $x = 0$ is not a local minimum or local maximum.

9/11 Post HW8