# Supply and Demand Curves

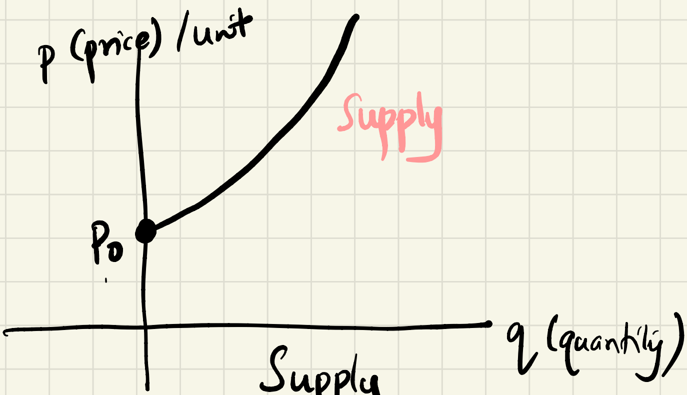
As the price increases, manufacturers are willing to supply more of the product.

Quantity demanded by the consumer falls.

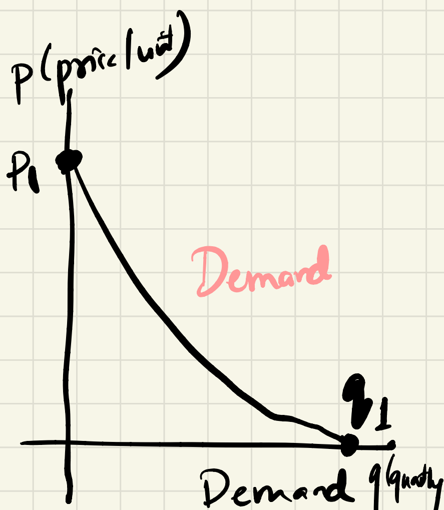
**Def.** The supply curve, for a given item, relates the quantity $q$ of the item that the manufacturers are willing to make per unit time to the price, $p$, for which the item can be sold.

The demand curve, relates $q$, the quantity of the item demanded by consumer per unit time to the price, $p$, of the item.

## ECONOMIST'S CONVENTION




Supply




Demand

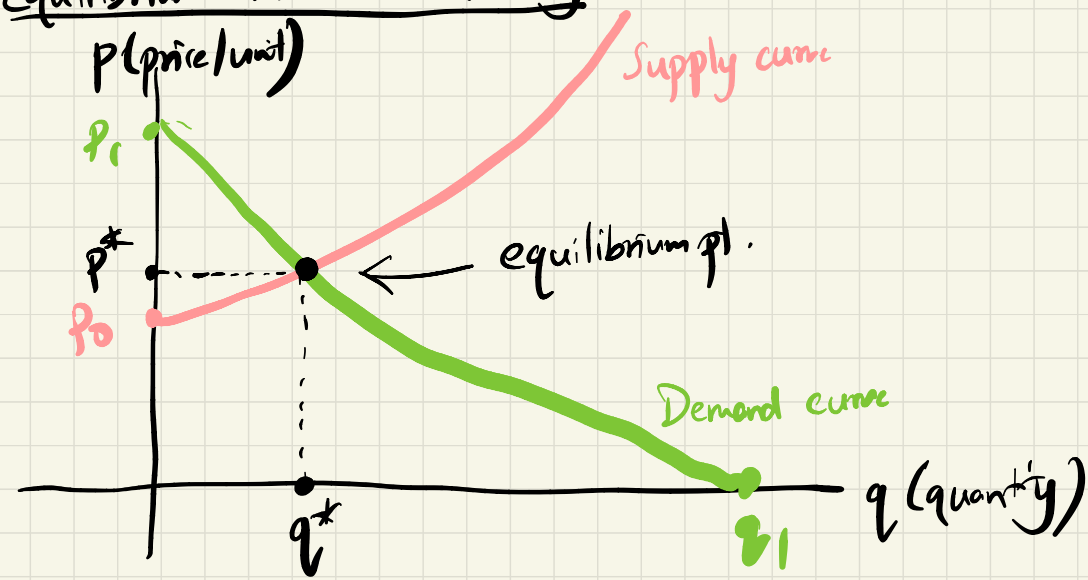
**Ex.** What is the economic meaning of $p_0$, $p_1$ and $q_1$.

**Soln.** $p_0$ is the price at which the quantity supplied is zero.

$p_1$ is the price at which the quantity demanded is zero.

$q_1$ is the quantity demanded if the price is zero.

## Equilibrium Price and Quantity




equilibrium point = $(q^*, p^*)$

Ex. Find the equilibrium price and quantity if

Quantity supplied = $3p - 50$

Quantity demanded = $100 - 2p$

Soln.

$$\text{Supply} = \text{Demand}$$

$$3p - 50 = 100 - 2p$$

$$3p + 2p = 100 + 50$$

$$5p = 150$$

$$p = \frac{150}{5} = 30$$

$$\boxed{p^* = \text{Equilibrium price} = \$30.}$$

Plug $p = 30$ into any of the two functions:

$$Q^* = 3 \cdot 30 - 50$$

$$= 90 - 50$$

$$= 40$$

$$\therefore \boxed{Q^* = \text{Equilibrium quantity} = 40}$$

# 1.5 Exponential Functions

## Example 1
### Population Growth

Population of Burkina Faso (millions)

| Year | Population | Change in population (millions) |
|---|---|---|
| 2007 | 14.235 | |
| | | 0.425 |
| 2008 | 14.660 | |
| | | 0.435 |
| 2009 | 15.095 | |
| | | 0.445 |
| 2010 | 15.540 | |
| | | 0.455 |
| 2011 | 15.995 | |
| | | 0.465 |
| 2012 | 16.460 | |
| | | 0.474 |
| 2013 | 16.954 | |

Notice that it's not linear because the change is not constant.

Divide each year's population by the previous year's population:

$$\frac{\text{Pop in 2008}}{\text{Pop. in 2007}} = \frac{14.660}{14.235} = 1.03$$

$$\frac{\text{Pop. in 2009}}{\text{Pop in 2008}} = \frac{15.095}{14.660} = 1.03$$

Population is increasing by a

| factor of 1.03 | percentage of 3 per year (3% per year) |
|---|---|

$$\text{Pop. after 1 year} = \text{Pop. currently} \times 1.03$$

$$\text{Pop. after 1 year} = \text{Pop. currenty} \times (1 + 0.03)$$

$$\boxed{\text{Pop. after 1 year} = \text{Pop. currently} + \underset{\text{current}}{\text{Pop.}} \times 0.03}$$

Whenever we have a constant percent increases 3% (here 3%) we have exponential growth.

Let $t$ be the no. of years since 2007.

When $t=0$, projected population is $14.235 = 14.235 \times (1.03)^0$

When $t=1$, projected population is $14.662 = 14.235 \times (1.03)^1$

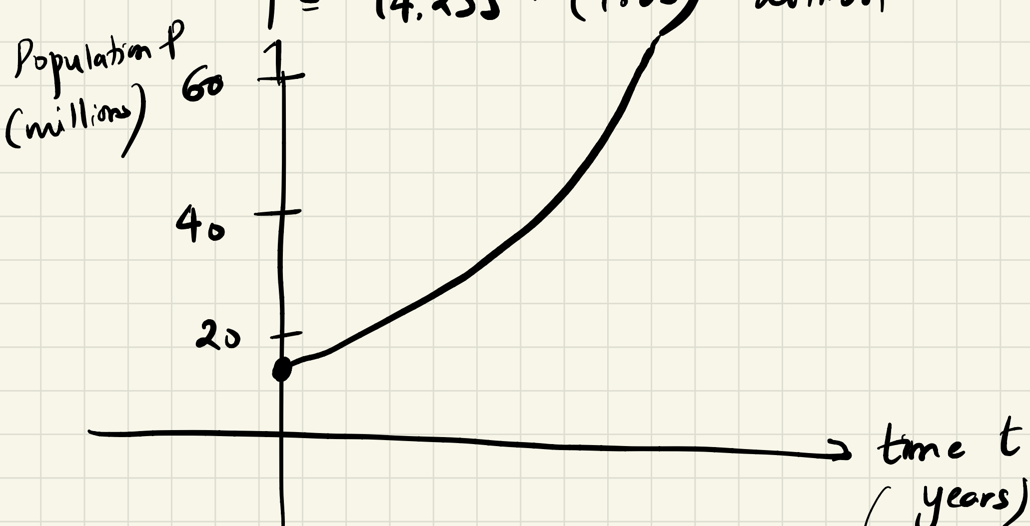
When $t=2$, projected population is $15.095 = 14.235 \times (1.03)^2$

When $t=3$, projected population is $15.535 = 14.235 \times (1.03)^2 \cdot (1.03)$

$$= 14.235 \times (1.03)^3$$

So, $P$, the population in millions, $t$ years after 2007, is approximated by

$$P = 14.235 \cdot (1.03)^t \text{ million}$$



## Example 2

### Elimination of a drug from a body

For the antibiotic ampicillin, approximately 40% of the drug is eliminated every hour. A typical dose of ampicillin is 250 mg. Suppose $Q = f(t)$, Q quantity of ampicillin (mg) in the bloodstream at time t hours since the drug was given.

At $t=0$ we have $Q = 250$

Since the quantity remaining at the end of each hour is 60% of the quantity remaining the hour before,

$$f(0) = 250$$

$$f(1) = 250 \cdot (0.6)^1$$

$$f(2) = 250\,(0.6)(0.6) = 250\,(0.6)^2$$

$$f(3) = 250\,(0.6)^2(0.6) = 250\,(0.6)^3$$

After t hours,

$$Q = f(t) = 250\,(0.6)^t$$

# General Exponential Function

Burkina Faso, growing at 3% per year

factor $a = 1 + 0.03 = 1.03$

Drug removal, decreasing at 40% per year

factor $a = 1 - 0.40 = 0.6$

We say $P$ is an exponential function of $t$ with base $a$ if

$$P = P_0 a^t$$

$P_0$ is initial quantity

$\underline{a}$ is factor by which $P$ changes when $t$ increases by 1 unit.

If $a > 1$, then we have exponential growth

If $0 < a < 1$, then we have exponential decay.

$$\boxed{a = 1 + r}$$

$r$ is decimal representatn of the percent rate of change

## Comparesion between linear and Exponential Functn

A linear function has constant rate of change

An exponential function has a constant relative rate of change

$$\left(\frac{P_1 - P_0}{P_0}\right)$$

**Problem 1** The amount of adrenaline in the body can change rapidly. Suppose the initial amount is 15 mg. Find a formula for A, the amount in mg at a time t minutes later if A is

a) Increasing by 0.4 mg per minute.

**Soln.** Ques. Is it linear or exponential?

Ans. Linear

$$\boxed{A = 0.4t + 15}$$

$$\boxed{y = mx + b}$$

slope

Initial value

b) Increase by 3% per minute.

Soln. Q. Is it linear or exponential?

Ans. Exponential.

Exponential function is given by

$$\boxed{P = P_0 a^t}$$

Initial value = 15mg

$a = ?$

$a = 1 + r$

$a = 1 + 0.03$

$= 1.03$

$$\therefore \boxed{A = 15(1.03)^t}$$

c) Decreasing by 0.4 mg per minute.

Soln. Linear

$$y = mx + b$$

Initial quantity = 15 mg

Slope (rate of chang) = $-0.4$

$$\boxed{A = -0.4t + 15}$$

d) Decreasing by 3% per minute

Soln. Exponential decay.

general function is $P = P_0 a^t$

Initial value = 15 mg

$$a = 1 - 0.03$$

$$= 0.97$$

$$\boxed{A = 15(0.97)^t}$$

HW 2 next week

Midterm 1 Feb. 9

Sample Test Problems Due Feb 9 midnight

Problem 2. $Q = f(t)$ is an exponential function of $t$.

If $f(20) = 88.2$

$f(23) = 91.4$

a) Find the base.

b) Find the percent growth rate

c) Evaluate $f(25)$.

Soln. (a) $f(t) = Q_0 a^t$

To find: Value of $a$.

$f(20) = 88.2$

$\Rightarrow Q_0 a^{20} = 88.2$ —— (i)

$f(23) = 91.4$

$\Rightarrow Q_0 a^{23} = 91.4$ —— (ii)

Divide eq (i) by eq (ii)

$$\frac{\cancel{Q_0} a^{20}}{\cancel{Q_0} a^{23}} = \frac{88.2}{91.4}$$

$$\frac{a^{20}}{a^{23}} = \frac{88.2}{91.4}$$

$$\frac{1}{a^{23-20}} = \frac{88.2}{91.4}$$

$$\frac{a^x}{a^y} = \frac{1}{a^{y-x}}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$\frac{1}{a^3} = \frac{88.2}{91.4}$$

$$a^3 = \frac{91.4}{88.2}$$

$$a = \sqrt[3]{\frac{91.4}{88.2}}$$

(cube roots on both sides)

$$\boxed{a = 1.012}$$

b)

$$a = 1 + r$$

$$1.012 = 1 + r$$

$$r = 1.012 - 1$$

$$= 0.012$$

$\therefore$ $\boxed{\text{percent growth rate is } 1.2\%}$

c) $f(25) = Q_0 (1.012)^{25}$

We don't know the value of $Q_0$.

Go back to equation (i)

$$Q_0 a^{20} = 88.2$$

$$Q_0 (1.012)^{20} = 88.2$$

$$\frac{Q_0 \cancel{(1.012)^{20}}}{\cancel{(1.012)^{20}}} = \frac{88.2}{(1.012)^{20}}$$

$$Q_0 = \frac{88.2}{(1.012)^{20}}$$

$$Q_0 = 69.5$$

$$f(25) = 69.5 (1.012)^{25}$$

$$\boxed{f(25) = 93.6}$$