HW 1 Due Monday Jan 25

Sample Test I will post the problems today.

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Legacy Platform

Course code: ↙ Course policy in Moodle

Sample Test - Take pictures of your work

upload to Moodle

Due on the midnight of the

midterm exams.

1.4. Applications of Functions to Economics

Def. The cost function, $C(q)$, gives the total cost of producing a quantity q of some good.

Remark: We expect C to be an increasing function.

Cost of production can be separated into two parts:

fixed costs - independent of quantity produced

variable costs - dependent on quantity produced

Example Let's consider a company that makes radio.

Fixed costs: factory and machinery, incurred even if no radios made

Variable costs: labor and raw material

The fixed costs for the company are \$24,000.
The variable costs are \$7 per radio.

$$\text{Total cost for company} = \text{Fixed costs} + \text{Variable costs}$$

$$= \$24{,}000 + 7 \cdot \text{No. of radios}$$

If q is the no. of radios produced,

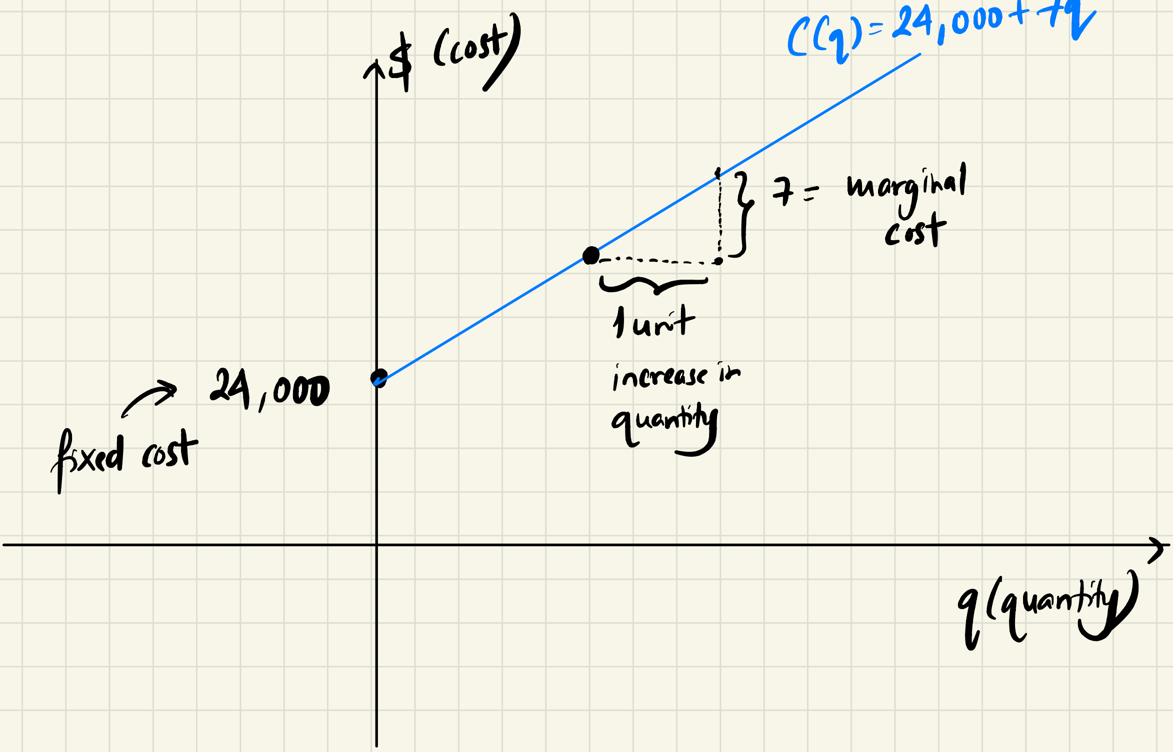
$$C(q) = 24{,}000 + 7q$$

<u>Def.</u> The variable cost for one additional unit is called the marginal cost. For a linear cost function, the marginal cost is the rate of change, or slope of the cost function.

Example 1 Graph the cost function $C(q) = 24{,}000 + 7q$.
Label the fixed costs and marginal cost.

Soln. $C(q) = 24{,}000 + 7q$

It is represented by a line whose slope is 7 and vertical intercept is 24,000.



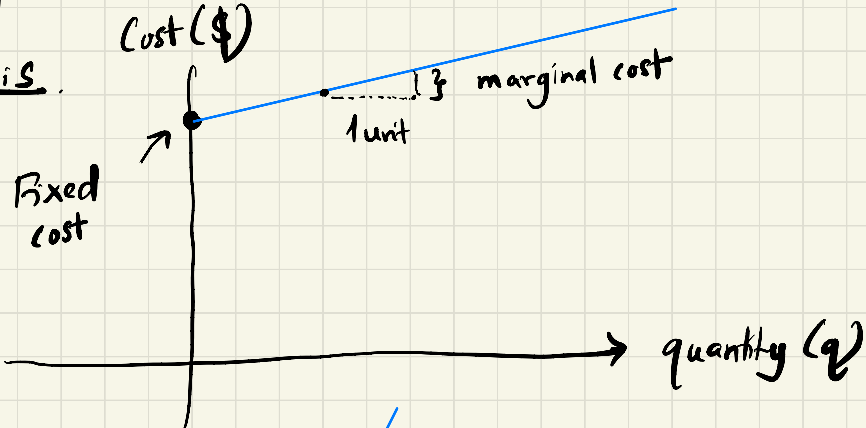
Problem 2 In each case, draw a graph of a linear cost function satisfying the given conditions.

a) Fixed costs are large but marginal cost is small.

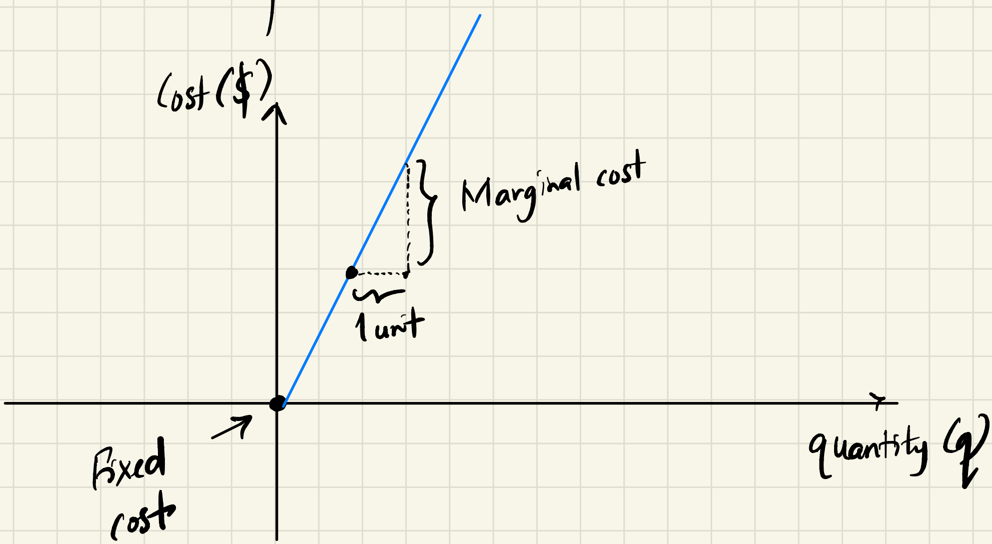
b) There are no fixed costs but marginal cost is high.

Try this.

a)



b)



The Revenue Function

Def. The revenue function, $R(q)$, gives the total revenue received by a firm from selling a quantity, q, of some good.

If the good sells at a price of p per unit, and the quantity sold is q, then

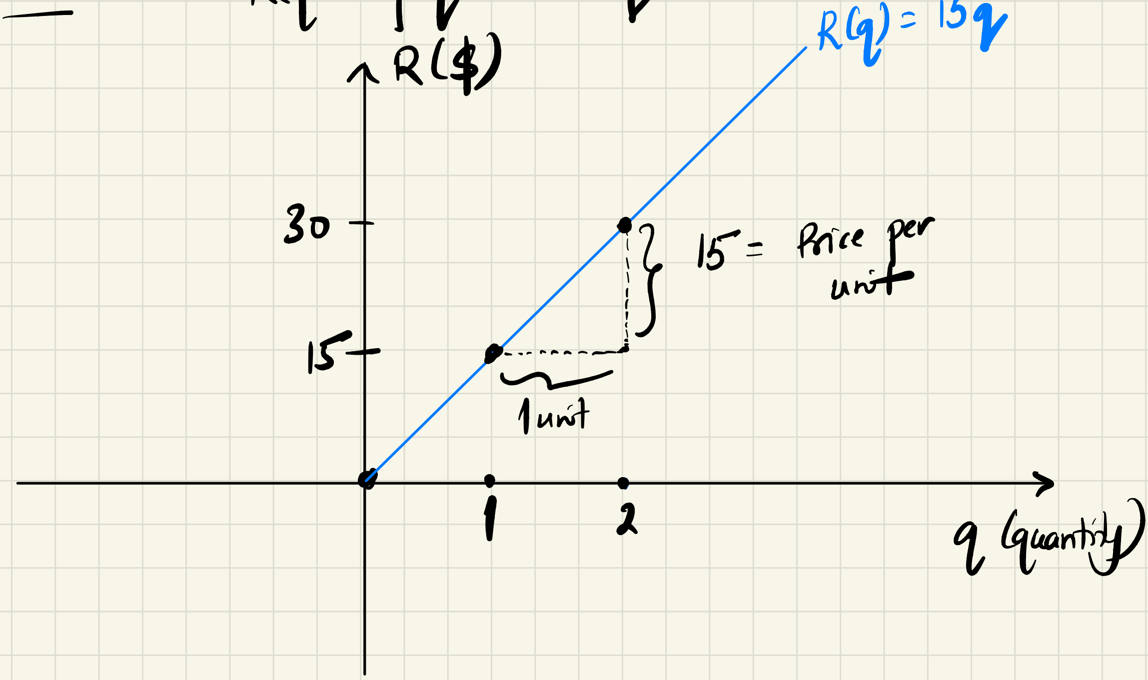
$$\text{Revenue} = \text{Price} \cdot \text{Quantity}$$

$$R = pq + \underline{0}$$

If the price does not depend on the quantity sold, so p is a constant, the graph of revenue function is a line through the origin, with slope equal to price p.

Example 3 If radios sell for \$15 each, sketch the manufacturer's revenue function. Show the price of a radio on the graph.

Soln. $R(q) = pq = 15q$



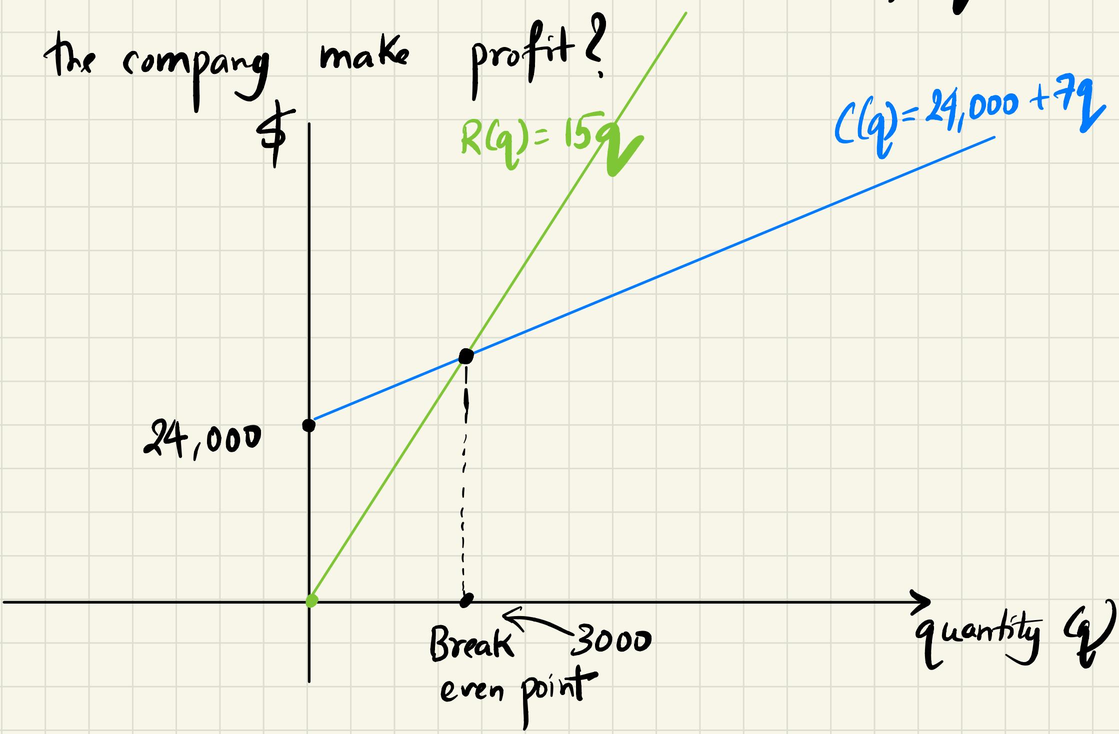
Example 4 Graph the cost function

$$C(q) = 24,000 + 7q$$ and

the revenue function

$$R(q) = 15q$$

on the same axes. For what values of q does the company make profit?



Ques. How do we find the value of the break even point?

Cost = Revenue

$$24{,}000 + 7q = 15q$$

$$24{,}000 = 8q$$

$$q = \frac{24{,}000}{8}$$

$$q = 3000$$

Company makes profit if it produces and sell more than 3000 radios.

The Profit Function

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\pi = R - C$$

<u>Def</u>. The break-even point is the point where profit is zero and revenue equals cost.

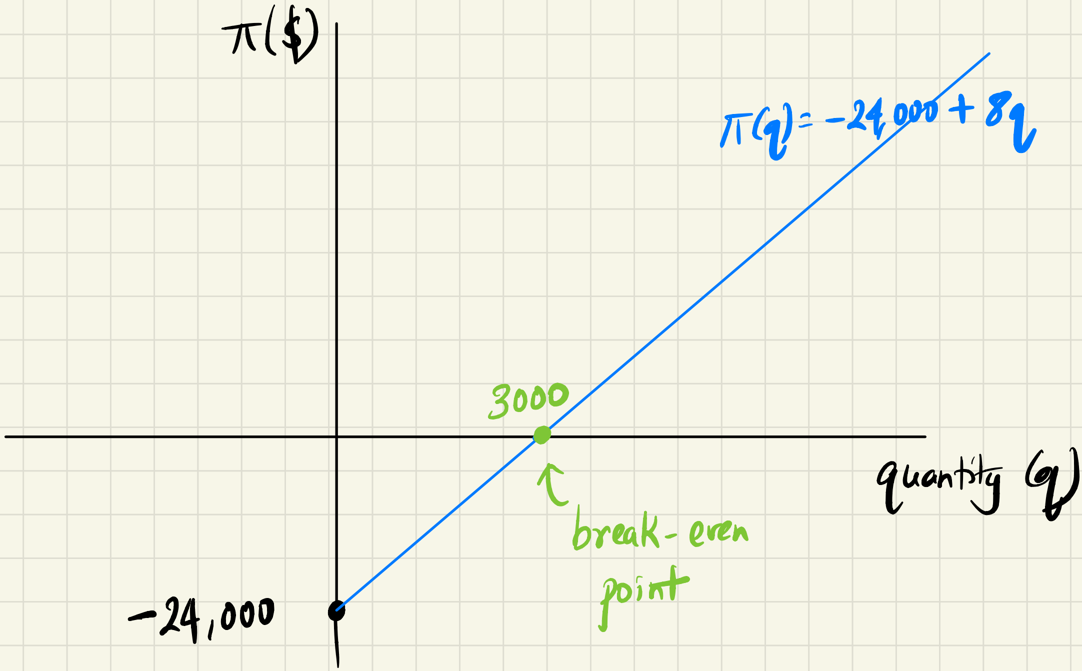
<u>Example 5</u> Find a formula for the profit function of the radio manufacturer. Graph it, marking the break-even point

<u>Soln.</u>

$$\pi(q) = R(q) - C(q)$$

$$= 15q - (24{,}000 + 7q)$$

$$= \underbrace{-24{,}000}_{\text{vertical intercept}} + \underbrace{8}_{\text{slope}}q$$



π($)
π(q) = -24,000 + 8q
3000
quantity (q)
break-even point
-24,000

Example 5

a) Using the table, estimate the break-even point for the company

q	500	600	700	800	900	1000	1100
$C(q)$	5000	5500	6000	6500	7000	7500	8000
$R(q)$	4000	4800	5600	6400	7200	8000	8800

Soln.

Ques 1. Between which two values of q ~~should~~ is the Break even point located?

Ans. Between 800 and 900.

Ques 2. Is it closer to 800 or 900? Why?

Ans. Closer to 800 because cost and revenue are closer at 800 than at 900.

$\boxed{\text{Break even point} = 830}$

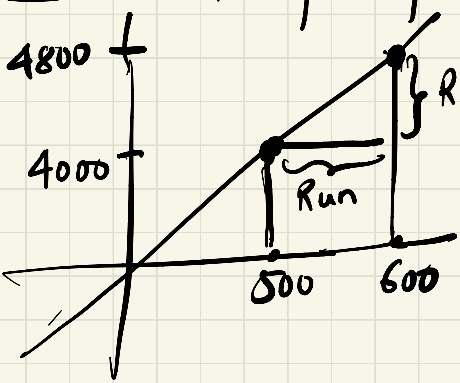
b) Find the company's profit if 1000 units are produced.

Soln.

$$\pi(1000) = R(1000) - C(1000)$$
$$= 8000 - 7500$$
$$= \$500$$

c) What price do you think the company is charging for it's product?

Soln. The price per unit = Slope of $R(q)$



$$= \frac{\text{Rise}}{\text{Run}} = \frac{4800 - 4000}{600 - 500}$$

$$= \frac{800}{100}$$

= 8 dollars per unit

Def. The marginal $\left\{\begin{array}{l}\text{cost}\\ \text{revenue}\\ \text{profit}\end{array}\right.$ is the slope of the $\left\{\begin{array}{l}\text{cost}\\ \text{revenue}\\ \text{profit}\end{array}\right.$ function.

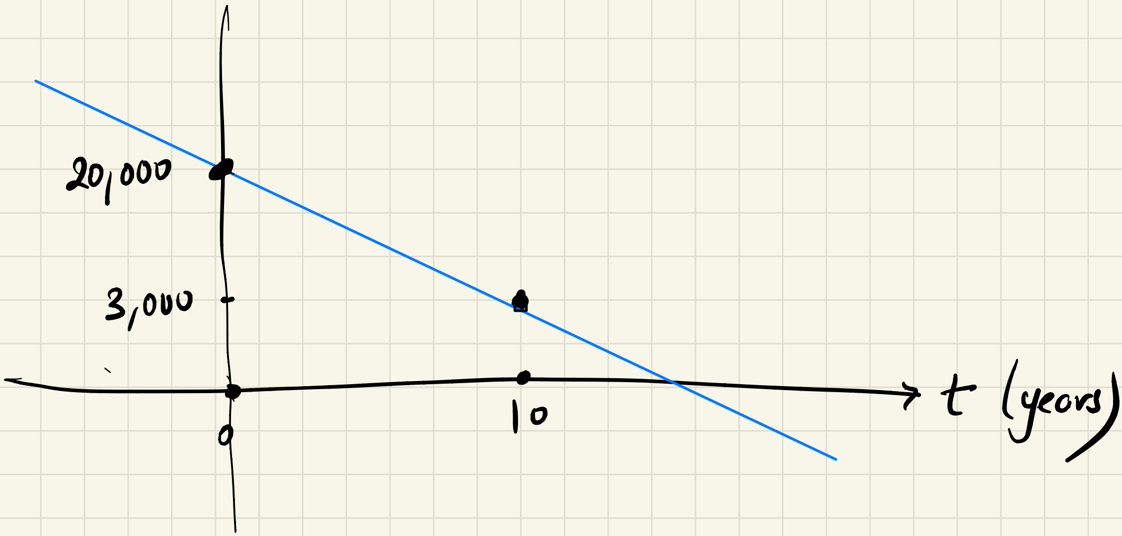
The Depreciation Function

Suppose that the radio manufacturer has a machine that costs $\$20,000$ and is sold ten years later for $\$3,000$. We say that the value depreciates from $\$20,000$ today to a resale of $\$3,000$ in ten years.

$V(t)$ is the function of number of years.

$$V(0) = 20,000$$

$$V(10) = 3,000$$



$$\text{Slope} = \frac{f(10) - f(0)}{10 - 0}$$

$$= \frac{3{,}000 - 20{,}000}{10}$$

$$= \frac{-17{,}000}{10} = -1700 \text{ dollars per year}$$

$$\boxed{y = mx + b}$$

This says that the value of machine is decreasing at a rate of \$1700 per year.

$$\boxed{V(t) = -1700t + 20{,}000}$$

Next time:

Supply and Demand Curve

Budget Restrictions

Taxes

Next class: Zoom

Poll on Tuesday to decide the Thursday meeting.