Midterm 2 March 4, Thursday

Covers Ch 2

HW 5 Due Thursday

HW 6 Due Friday, March 5

Sample Test Problem Due March 4,
Midnight

(I will post the selected problems in Moodle)

2.3 Intrepretations of the Derivative

Leibniz notation

Let $y = f(x)$ be a function.

Let a be a number.

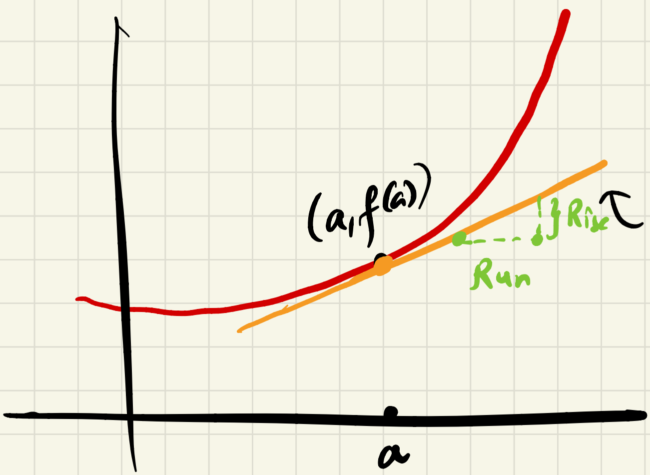
$$f'(a) = \left.\frac{dy}{dx}\right|_{x=a}$$ ← Leibniz notation

2 interpretations of $f'(a)$:

1. Limit of the average rate of changes as the run Δx becomes smaller and smaller.

   Special case: Instantaneous velocity

2. Slope of the tangent at $(a, f(a))$.



Slope = $\frac{\text{Rise}}{\text{Run}}$

slope of this line $= f'(a) = \left.\frac{dy}{dx}\right|_{x=a}$

Relative Rate of Change

Def. Relative rate of change of $y = f(t)$ at $t = a$

$$= \frac{f'(a)}{f(a)}$$

Memorize

$$= \frac{\left.\frac{dy}{dt}\right|_{t=a}}{f(a)}$$

Problem 1 Annual world soybean production, $W = f(t)$, in million tons, is a function of t years since the start of 2000.

a) Interpret the statements $f(8) = 253$ and $f'(8) = 17$ in terms of soybean production.

Soln. $f(8) = 253$

In 2008, the annual world soybean production was 253 million tons.

$f'(8) = 17$

In 2008, the annual soybean production was increasing at the rate of 17 million tons per year.

b) Calculate the relative rate of change of W at $t=8$; interpret it in terms of soybean production.

Soln. Relative rate of change of soybean productn at $t=8$ $= \dfrac{f'(8)}{f(8)}$

$= \dfrac{17}{253}$

≈ 0.067 } in decimals

$0.067 \times 100 = 6.7\%$

In 2008, the soybean production was increasing at the continuous rate of 6.7% per year.

2.4. Second Derivative

For a function f, the derivative of its derivative is called the ==second derivative==, written f''

$$\underbrace{f \longrightarrow f' \longrightarrow f''}_{\text{functions}}$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad f'$

Leibniz notation: $f'' = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)$

$$= \frac{d^2y}{dx^2}$$

$f''(a)$ is written as $\left.\dfrac{d^2y}{dx^2}\right|_{x=a}$

Recall:

$f' > 0$ on interval, f is increasing on that interval.

$f' < 0$ on interval, f is decreasing on that interval.

So, (Follow from the previous statements)

$f'' > 0$ on interval, f' is increasing on that interval.

$f'' < 0$ on interval, f' is decreasing on That interval.

We can say more:

$f'' > 0$ on interval, f' is increasing and f is concave up

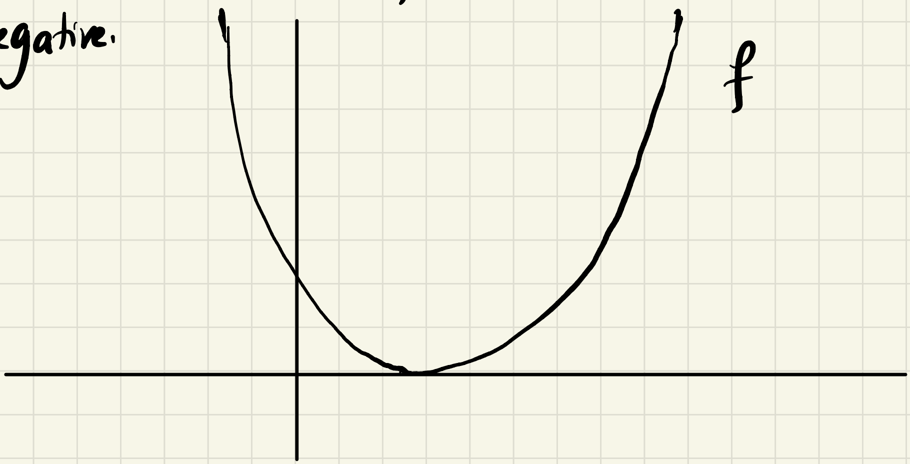
$f'' < 0$ on interval, f' is decreasing and f is concave down.

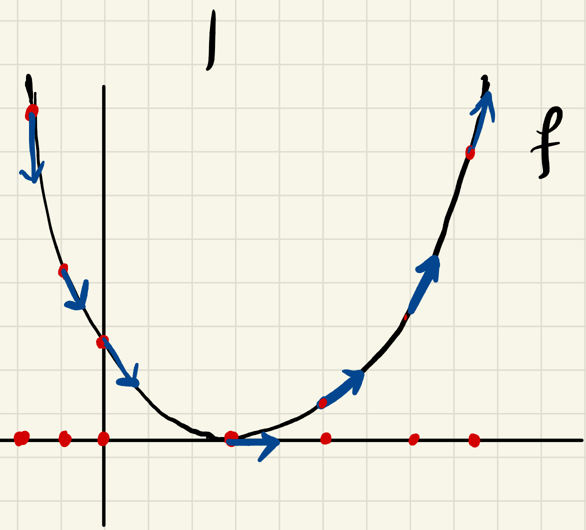
Memorize

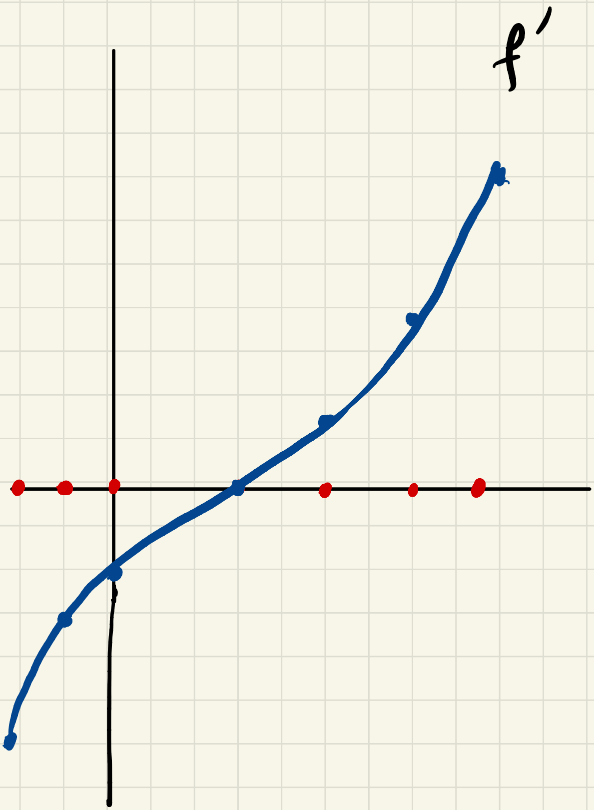
Problem 1

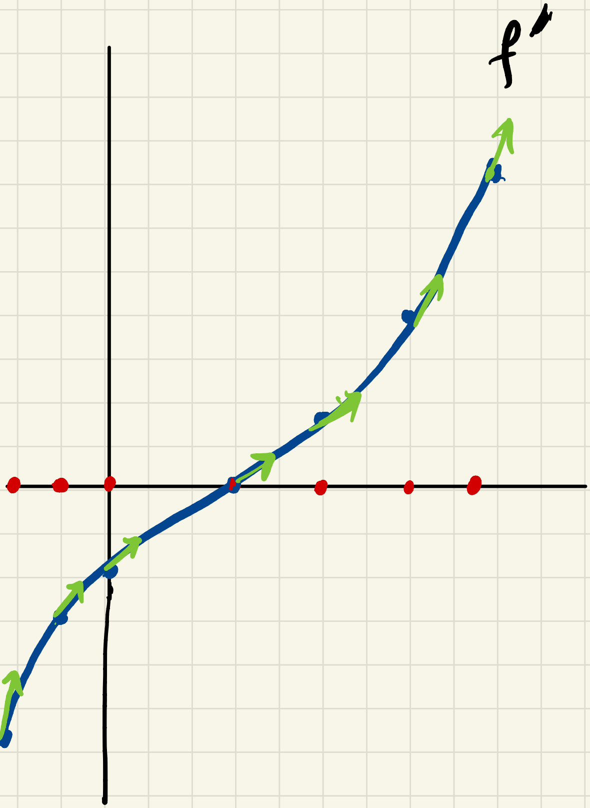
For the functions below decide where their second derivatives are positive and where they are negative.

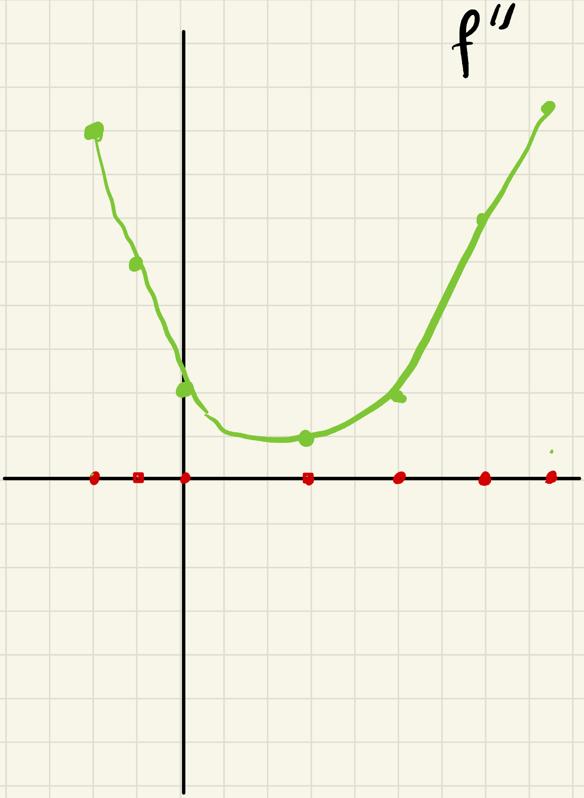
a)



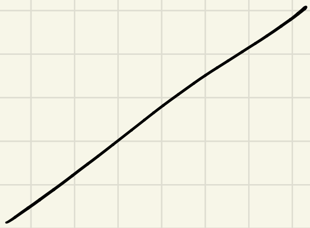




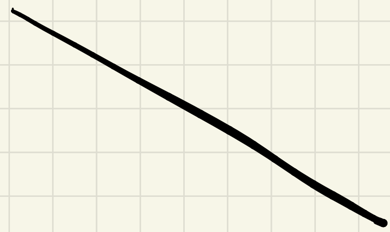




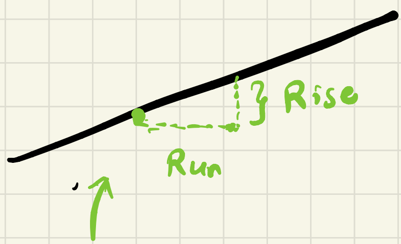
f'' is positive everywhere.



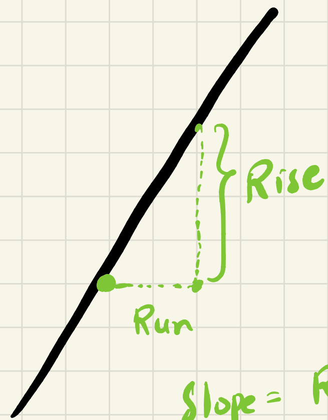
positive slope



negative slope



vs.



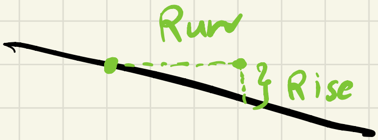
$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

Both have positive slope

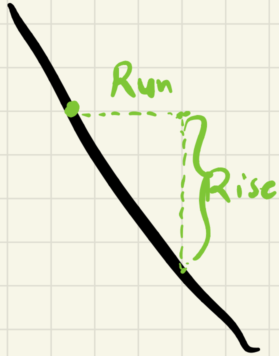
Run is same both lins

Second line has bigger slope



$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

vs.



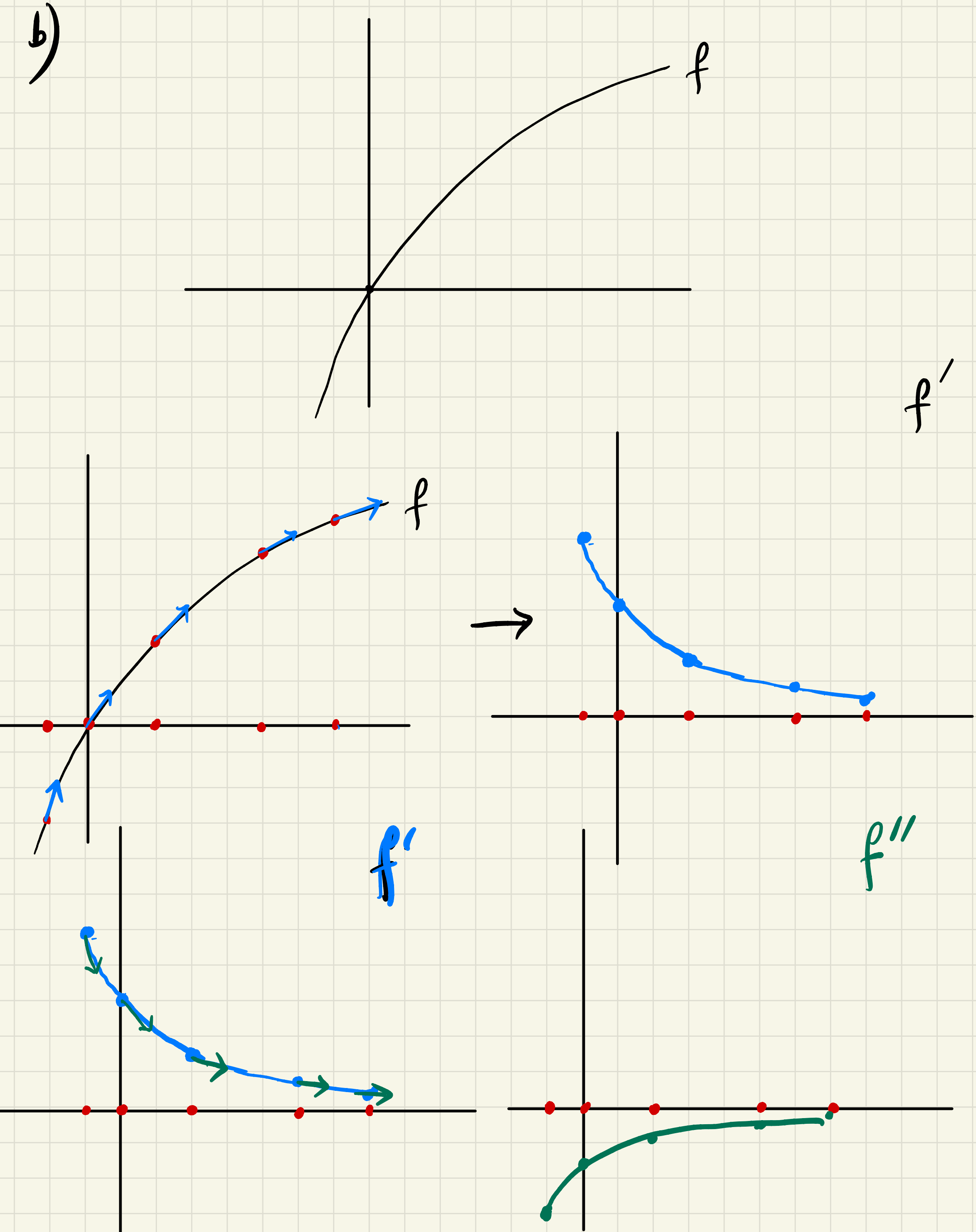
$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$

Both have negative slope.

Second line has slope with bigger magnitude.

-2 vs. -7

b)



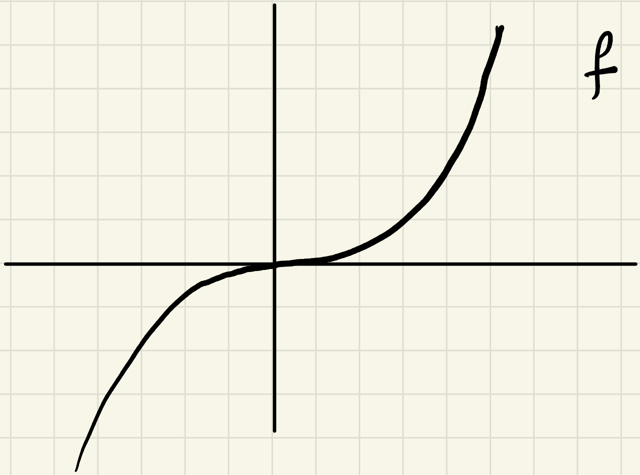
f'' is negative everywhere.

Alternative:

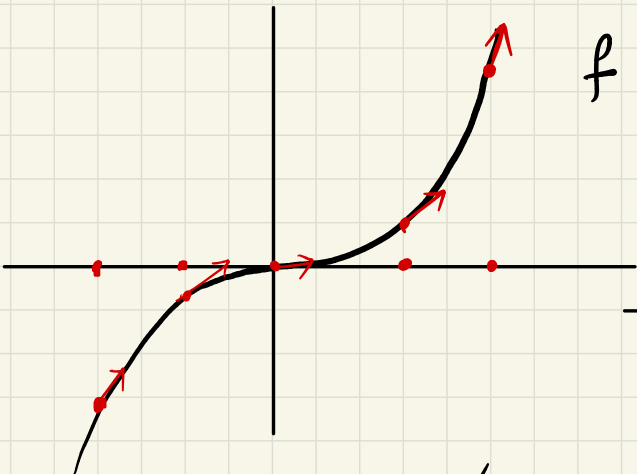
Since f' is decreasing and f is concave down, $f'' < 0$ everywhere

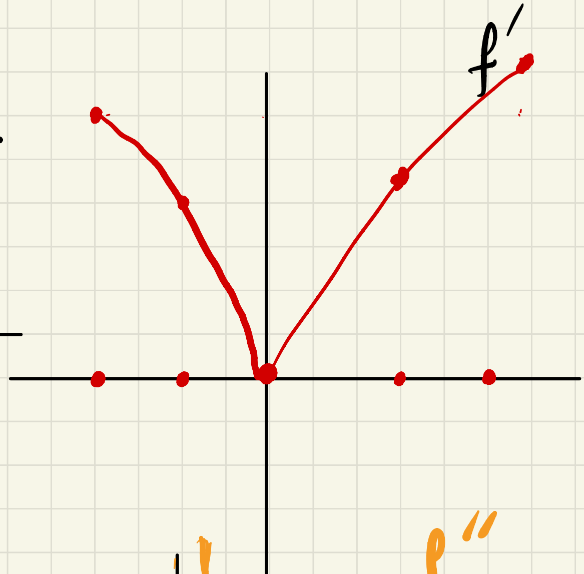
From the facts in PG. 7

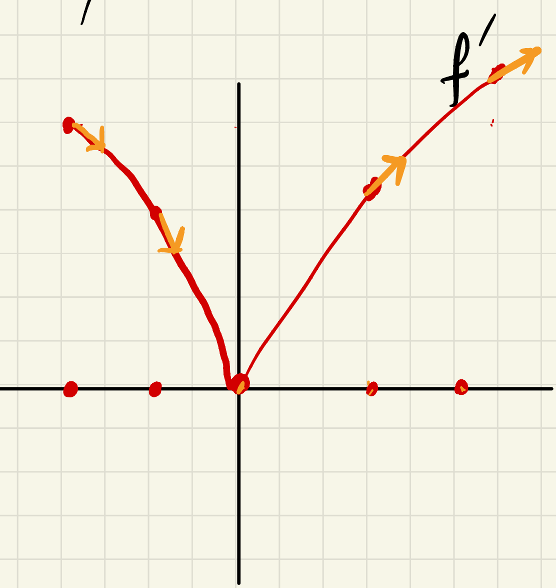
c.

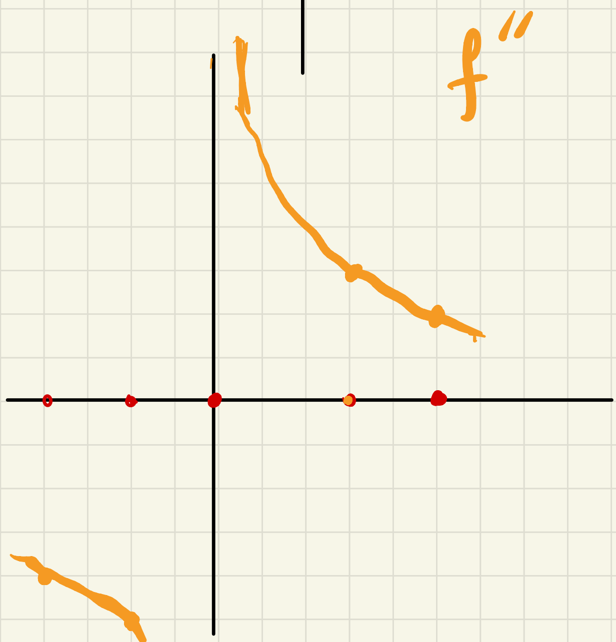


Slope
$-\infty$









$f'' < 0$ for $x < 0$

$f'' > 0$ for $x > 0$

Alternative solution:

For $x < 0$ f' is decreasing

f is concave down

For $x > 0$ f' is increasing

f is concave up

From the facts in PG 7 we get
The same conclusion.