- Your grades are posted on Moodle.
- Next class in person

  You can collect your midterm next class.
- 4 midterms 50%

| | |
|---|---|
| 4 midterms | 50% |
| Homework | 30% |
| Sample Test | 5% |
| Final | 15% |
| | 100% |

- I replace your lowest midterm score with the final exam score.
- HW 4 Due Tonight
  - Ask me if you need an extension.

Recall:

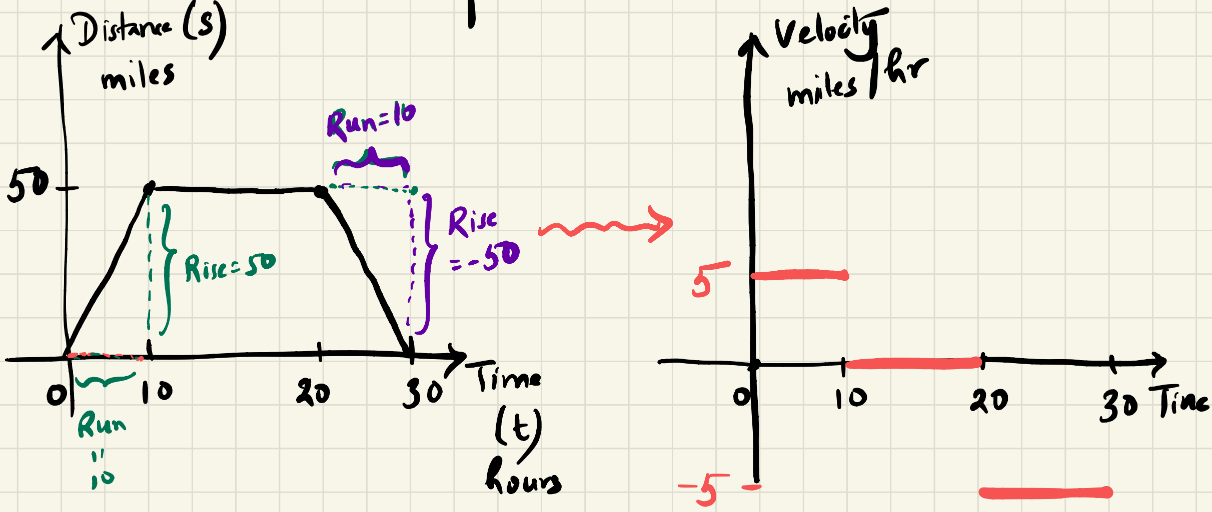
$$\text{Average velocity} = \frac{\Delta s}{\Delta t} \begin{array}{l} \leftarrow \text{distance travelled} \\ \leftarrow \text{time taken} \end{array}$$

$$\text{Instantaneous velocity} = \frac{\Delta s}{\Delta t} \qquad \Delta t \text{ is very small}$$

The instantaneous velocity is the limit of the average velocities over shorter and shorter intervals.

(The smaller the value of $\Delta t$, the more precise the value of the instantaneous velocity.)

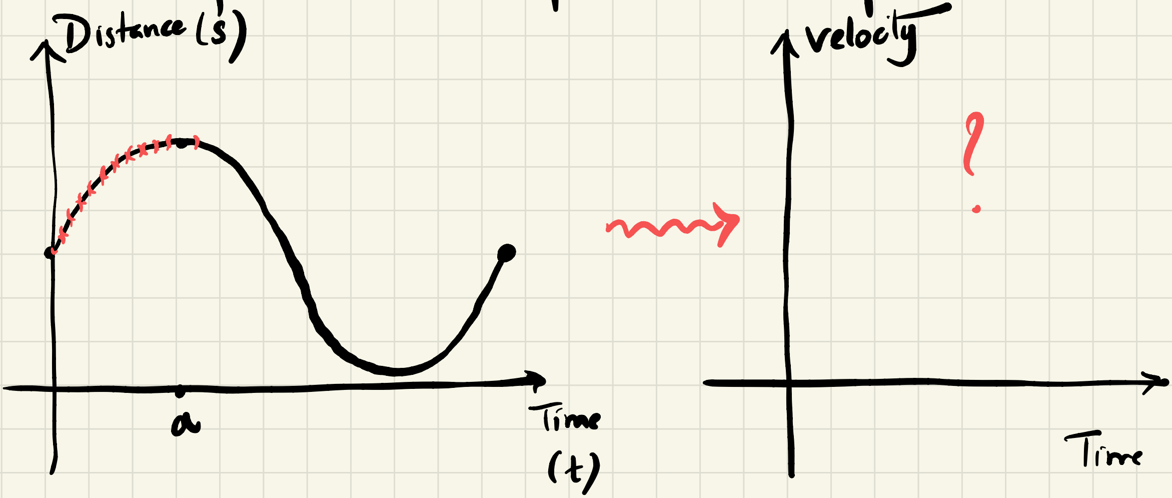
Distance function $\xrightarrow{\text{Slope at each point}}$ Velocity function




$\frac{50}{10} = 5 \qquad \frac{\text{Rise}}{\text{Run}} = \frac{-50}{10} = -5$

We know how to find the velocity function from the distance function if the graph of the distance function is piecewise linear (composed of a bunch of lines).

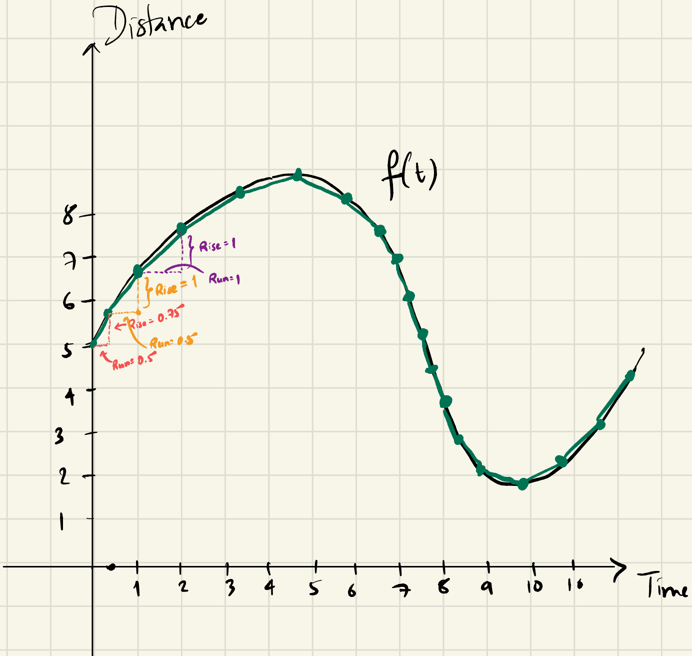
What if the distance function is not piecewise linear?

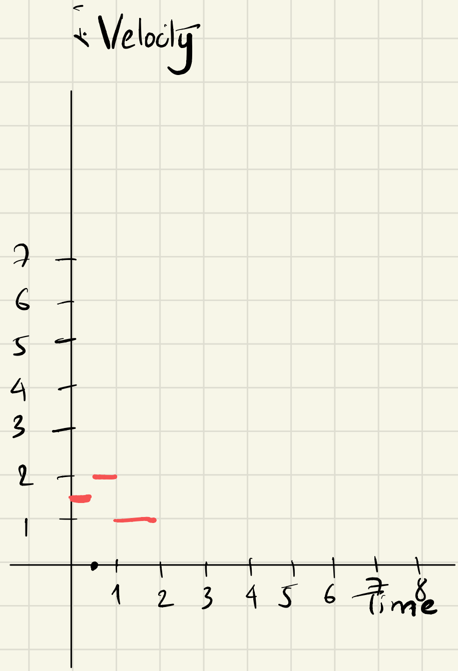



Notice that if you zoom into the graph, the small portion looks almost like a line.

- Break up the graph into very small pieces such that each piece looks almost like a line.
- We can approximate our graph by a piecewise linear function.
- Take slopes of each portion.

(This is just to give you an idea behind derivatives. It is not rigorous.)



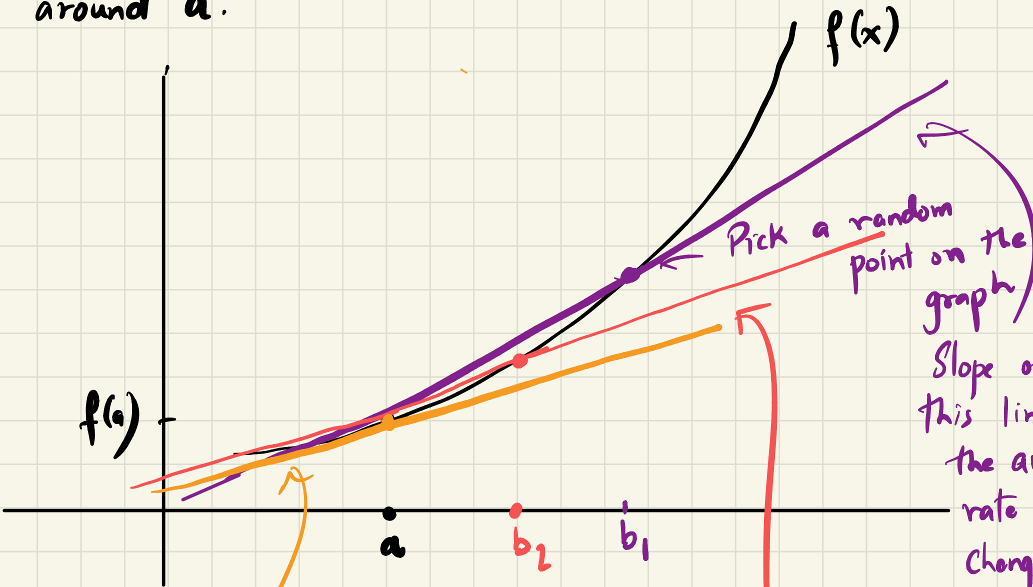

Green graph is piecewise linear composed of line
It is just an approximation of f(t) by a piecewise linear function

$\frac{Rise}{Run} = \frac{0.75}{0.5} = 1.5$ $\frac{Rise}{Run} = \frac{1}{0.5} = 2$ $\frac{Rise}{Run} = \frac{1}{1} = 1$

Eventually, the velocity function will be continuous,
Although, it is not continuous for a piecewise linear function.

**Def.** The derivative of $f$ at $a$, written as $f'(a)$, is defined to be the instantaneous rate of change of $f$ at $a$.

The instantaneous rate of change of $f$ at $a$, is defined to be the limit of the average rate of change of $f$ over shorter and shorter intervals around $a$.




Slope of this line is the average rate of change from $a$ to $b_1$

Slope is the avg. rate of change from $a$ to $b_2$

Eventually as the $b$'s get closer and closer to $a$, you will the tangent line to the graph.

Derivative at $a$ = $f'(a)$ = Slope of this tangent.

$$g(x) = 3^x$$

$$g'(1)$$

Avg. rate of change around 1 where your run is very small.

Avg. rate of change from 1 to 1.01 $= \dfrac{g(1.01) - g(1)}{1.01 - 1}$

$$= \frac{3^{1.01} - 3^1}{0.01}$$

$$= 3.3140$$

This is not exactly the derivative $g'(1)$

But it is very close to it.

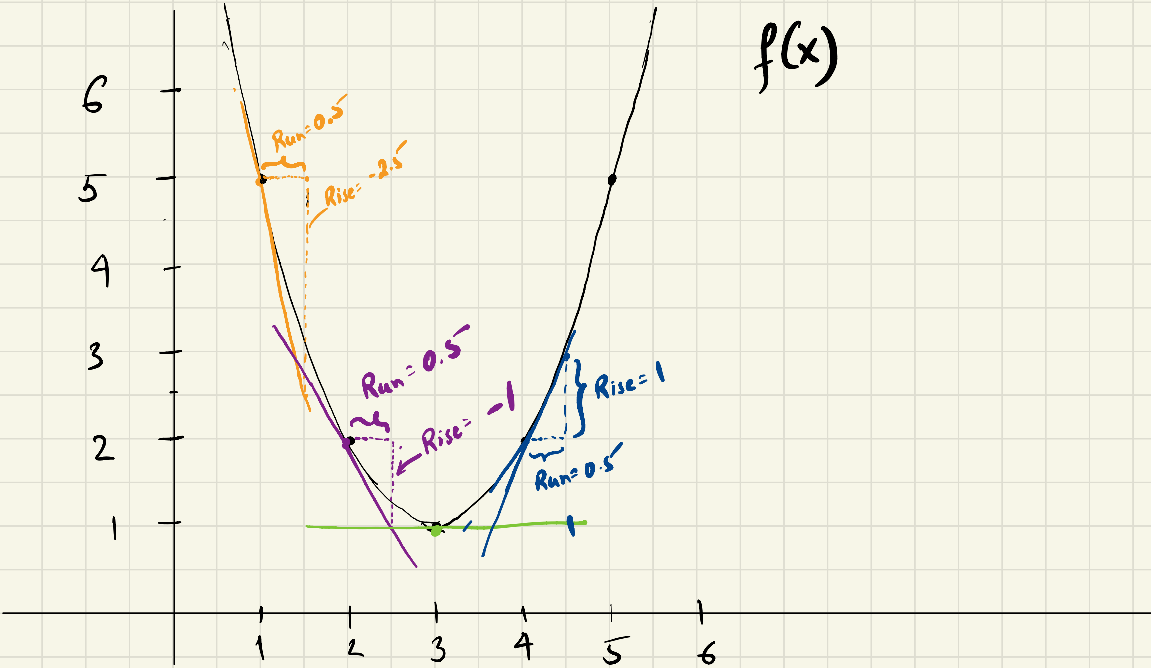
Av. rate of change from 1 to 1.001 $= \dfrac{g(1.001) - g(1)}{1.001 - 1}$

$$= \frac{3^{1.001} - 3^1}{0.001}$$

$$= 3.2976$$

**Problem 1** $f(x)$ is given.

Estimate $f'(1)$, $f'(2)$, $f'(3)$, $f'(4)$.




$f'(1)$ = Slope of the tangent at $(1,5)$

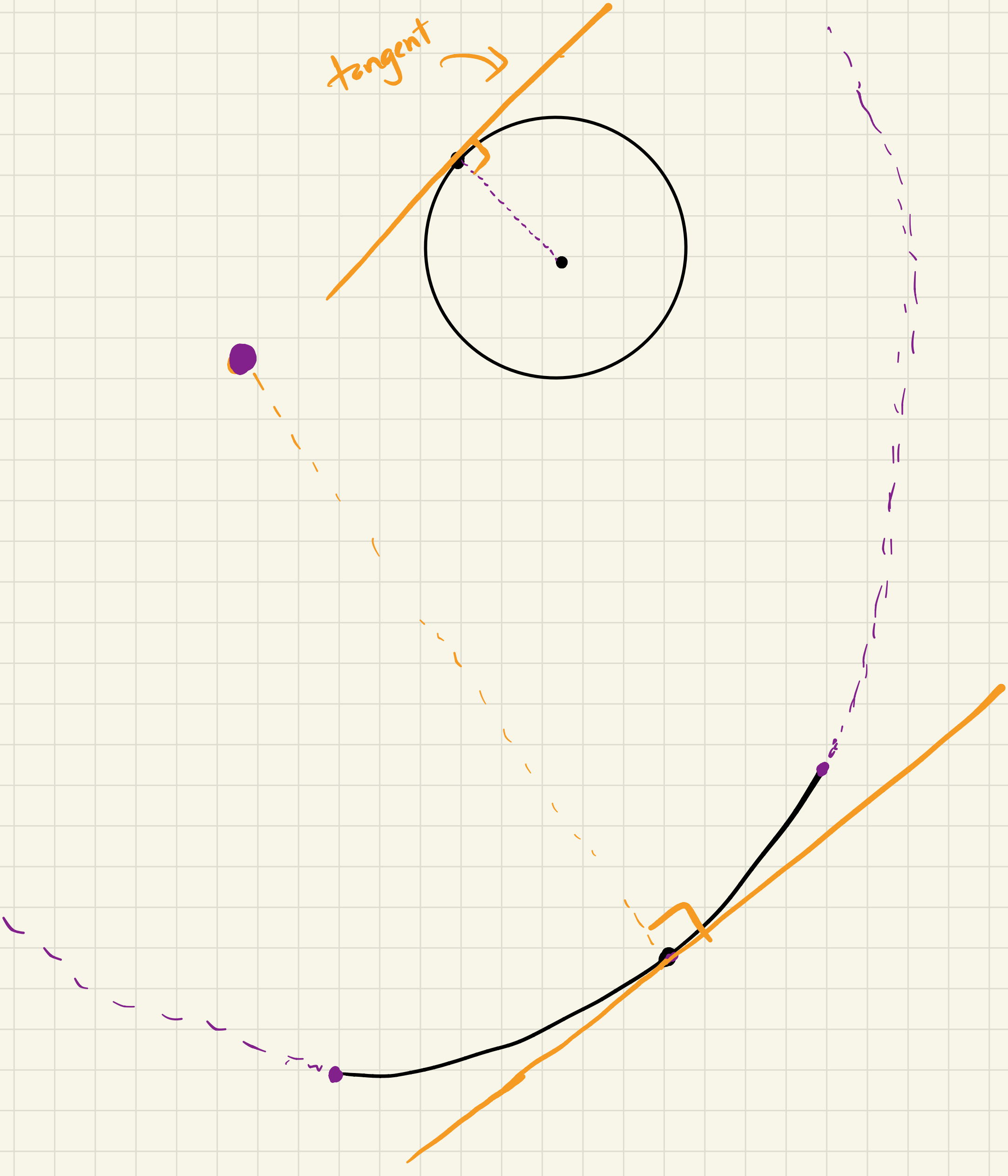
$= \dfrac{\text{Rise}}{\text{Run}} = \dfrac{-2.5}{0.5} = \boxed{-5}$

$f'(2)$ = Slope of the tangent at $(2,2)$

$= \dfrac{\text{Rise}}{\text{Run}} = \dfrac{-1}{0.5} = \boxed{-2}$

$f'(3) = \boxed{0}$

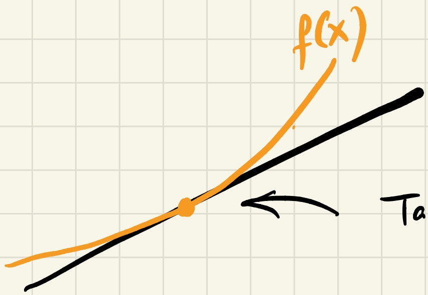
$f'(4) = \dfrac{\text{Rise}}{\text{Run}} = \dfrac{1}{0.5} = 2.$



tangent

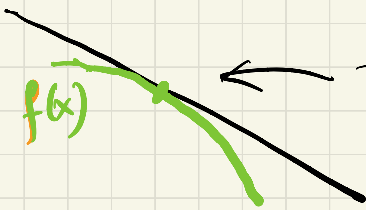
What does the Derivative tell us about the function?

$f' > 0$ on an interval, $f$ is <u>increasing</u>.



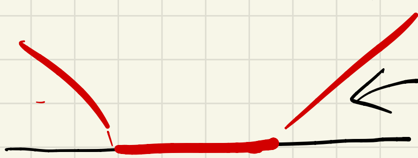

Tangent has positive slope

$f' < 0$ on an interval, $f$ is <u>decreasing</u>.



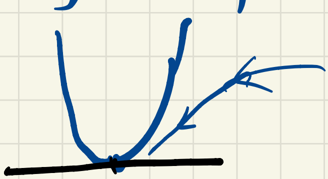

Tangent has negative slope

$f' = 0$ on an interval, $f$ is constant



Tangent has 0 slope on the interval

$f' = 0$ only at a point does not imply that $f$ is constant



Here $f'$ is zero but $f$ is not constant.

Magnitude of the derivative gives us the magnitude of the rate of change of $f$. If $f'$ is large, $f$ is steep – up if $f'$ is positive

– down if $f'$ is negative.

If $f'$ is small, the graph of $f$ is gently sloping