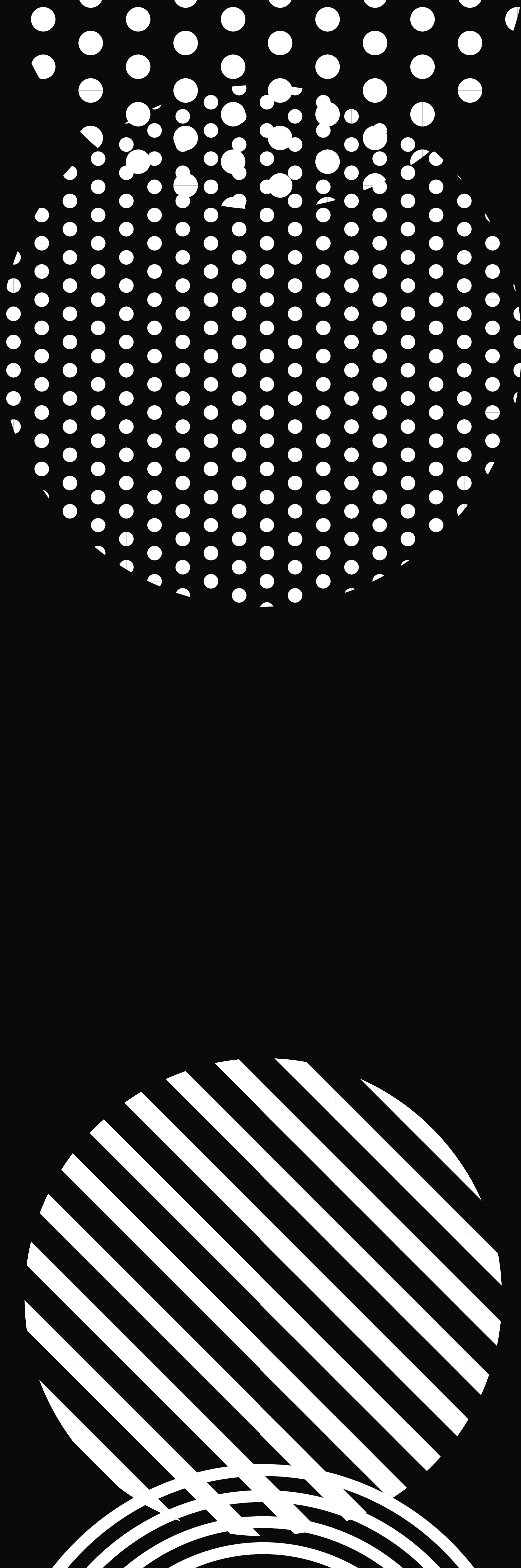


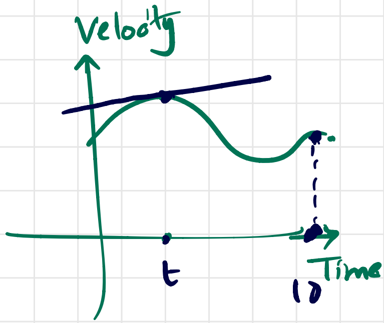
- Grades have been posted
- Midterm 4 April 27, Tuesday

Ch. 5 Accumulated Change: The Definite Integral

Calculus

Derivative Integral

Fundamental Theorem of Calculus



Concrete case: Velocity, Distance, Time.

Derivative = Instantaneous velocity.

Integral = Distance Travelled

Recall:

$$\text{Distance} = \text{Velocity} \times \text{Time}$$

or

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

$$\boxed{\text{But, only works when velocity is constant.}}$$

Ex. 70 miles / hr

3 hrs

$$\text{Distance} = 70 \text{ miles/hr} \cdot 3 \text{ hrs}$$

$$= 210 \text{ miles.}$$

Example 1 Suppose a car is moving with increasing velocity and we measure the car's velocity every two seconds.

Time (sec)	0	2	4	6	8	10
Velocity (ft/sec)	20	30	38	44	48	50

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Ques: How far has the car travelled?

We don't know how fast the car is moving at every moment (i.e. we don't have the whole velocity vs. time graph)

So we cannot calculate the distance exactly. But we can make an estimate.

1st two seconds: $[0, 2]$

The velocity is increasing. So, the car is going at least 20 ft/sec.

$$20 \text{ ft/sec} \cdot 2 \text{ sec} = 40 \text{ ft}$$

Car goes at least 40 ft in the first two seconds.

2nd two seconds: $[2, 4]$

The car is moving at at least 30 ft / sec.

$$30 \text{ ft/sec} \cdot 2 \text{ sec} = 60 \text{ ft}$$

The car moves at least 60 ft in the 2nd two seconds.

. - - - .

$$20 \cdot 2 + 30 \cdot 2 + 38 \cdot 2 + 44 \cdot 2 + 48 \cdot 2$$
$$= 360 \text{ feet}$$

This is an underestimate of the total distance travelled

To get an overestimate:

1st two seconds: $[0, 2]$

Assuming a constant velocity of 30 ft / sec.

$$\text{distance} = 30 \text{ ft/sec} \cdot 2 \text{ sec} = 60 \text{ feet}$$

The car moves at most 60 feet in the 1st two seconds.

2nd two seconds: $[2, 4]$

$$\text{distance} = 38 \text{ ft/sec} \cdot 2 \text{ sec} = 76 \text{ feet}$$

The car moves at most 76 feet in 2nd two seconds.

Add them up:

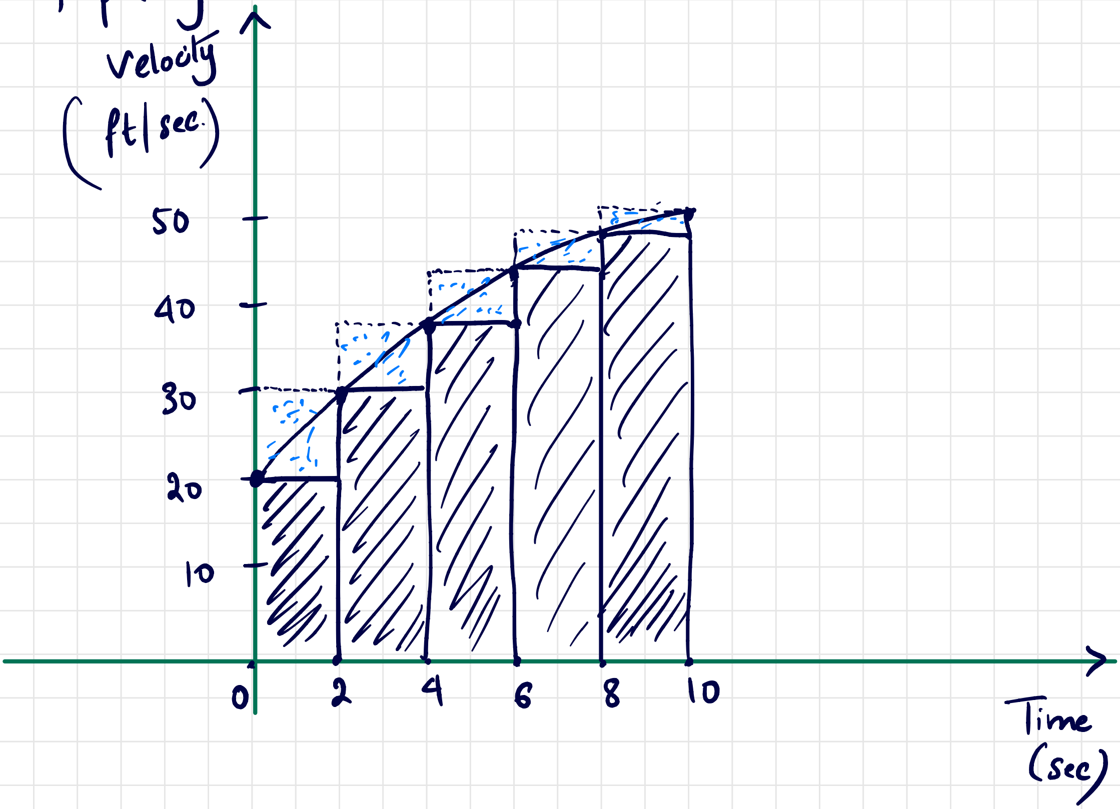
$$30 \cdot 2 + 38 \cdot 2 + 44 \cdot 2 + 48 \cdot 2 + 50 \cdot 2$$

$$= 420 \text{ feet.}$$

$$360 \text{ feet} < \text{Distance Travelled} < 420 \text{ feet}$$

Better estimate: $\dfrac{360 + 420}{2} = \boxed{390 \text{ feet.}}$

Graphically:



Underestimate

$= 20\cdot 2 + 30\cdot 2 + 38\cdot 2 + 44\cdot 2 + 48\cdot 2$

= Area of rectangles

Overestimate

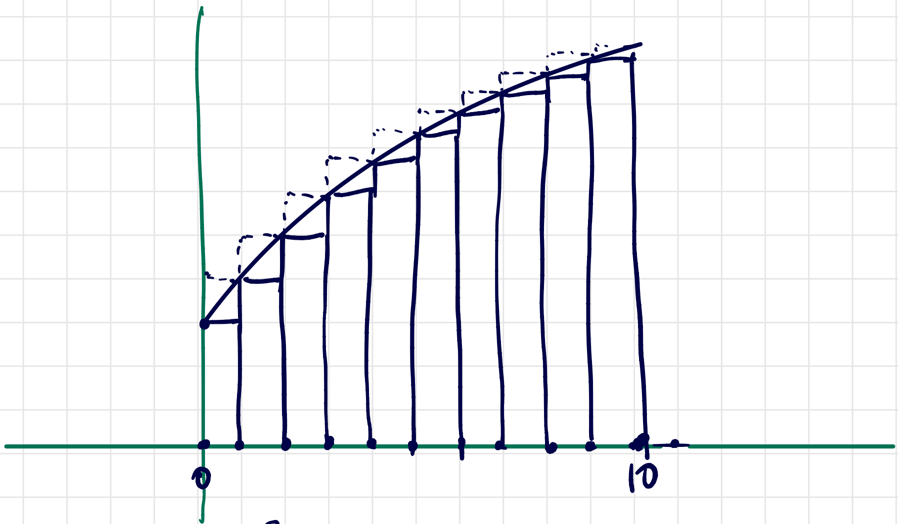
$= 30\cdot 2 + 38\cdot 2 + 44\cdot 2 + 48\cdot 2 + 50\cdot 2$

= Area of rectangle + rectangles.

Newton/Leibniz observation:

As you go on increasing the number of time measurements, i.e. we increase the no. of subdivisions: every second, every half second, etc...

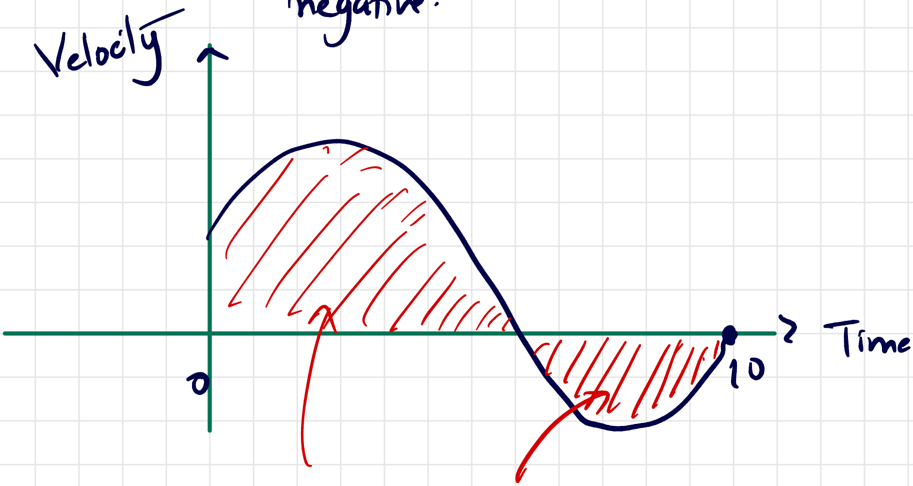
The overestimate and underestimate get closer and closer to the area under the velocity graph.



[Newton/Leibniz] Theorem

The total distance travelled is the area under the velocity curve.

Warning Velocity can be negative. In that case the area, distance travelled will be negative.



area positive + area negative = Distance travelled.

Problem1

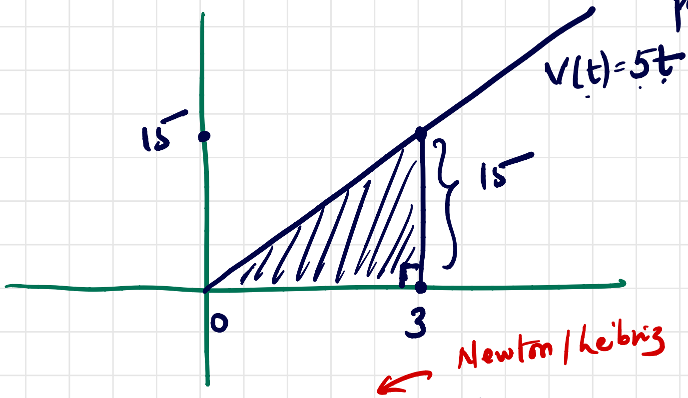
Time t in seconds.

The velocity of bicycle, in feet per second, is given by $v(t) = 5t$.

How far does the bicycle travel in the first 3 seconds after $t = 0$?

Soln $\quad v(t) = 5t$

Notice this is a linear function (line slope is 5 passes through origin)



Newton / Leibniz

Distance travelled from 0 to 3 sec $=$ Area under the graph from 0 to 3

$= $ Area of right triangle

$= \frac{1}{2} \cdot \text{base} \cdot \text{height}$

$= \frac{1}{2} \cdot 3 \text{ sec} \cdot 15 \text{ feet/sec}$

$= \boxed{22.5 \text{ feet}}$

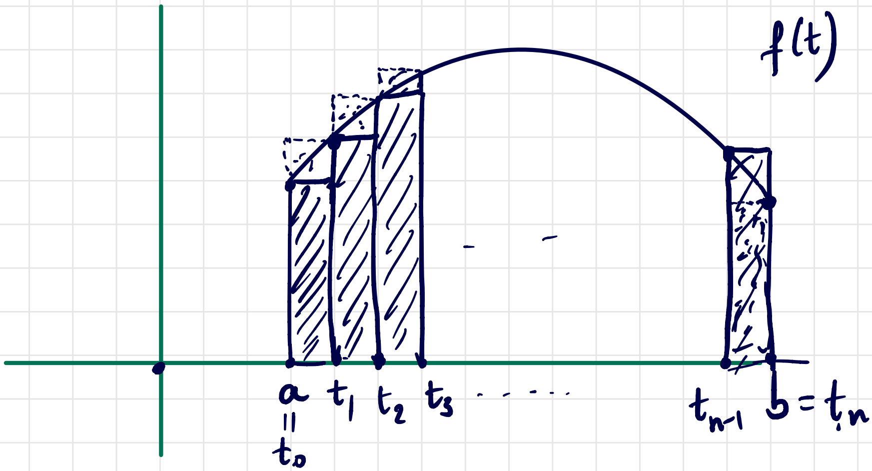
<u>Note:</u>

This idea works with rate of change of any function, not just the velocity function.

See Ex. 2 Sec 5.1.

Left and Right Hand Sums

$f(t)$ continuous for $a \le t \le b$



We divide the interval $[a, b]$ into n equal subdivisions each of width

$$\Delta t = \frac{b-a}{n} \quad \begin{array}{l} \leftarrow \text{length of } [a,b] \\ \leftarrow \text{no. of parts} \end{array}$$

Left hand sum $= f(t_0)\Delta t + f(t_1)\Delta t + f(t_2)\Delta t + f(t_{n-1})\Delta t$

Right hand sum = $f(t_1)\Delta t + f(t_2)\Delta t + f(t_3)\Delta t + \dots + f(t_n)\Delta t$

$t_0 \quad t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7 \quad t_8 \quad t_9 \quad t_{10}$

$t_0 = a$

Left hand sum = $f(t_0)\Delta t + f(t_1)\Delta t + \dots + f(t_9)\Delta t$

<u>Sigma notation:</u>

$\sum$ Greek letter capital sigma

Summation.

end

$$\sum_{i=\square}^{\square} f(i)$$

index

start

$$\sum_{i=0}^{3} i^2 = 0^2 + 1^2 + 2^2 + 3^2 = 14$$

$$\sum_{i=1}^{4} i/2 = \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2}$$

Left Hand sum

$$= \sum_{i=0}^{n-1} f(t_i)\,\Delta t$$

$$= f(t_0)\Delta t + f(t_1)\Delta t + \dots + f(t_{n-1})\Delta t$$

Right Hand sum

$$= \sum_{i=1}^{n} f(t_i)\,\Delta t$$

– Will post a hw. today