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Midterm 4 Tuesday April 27 (WileyPlus, Class Time)

Sample Test Due April 27

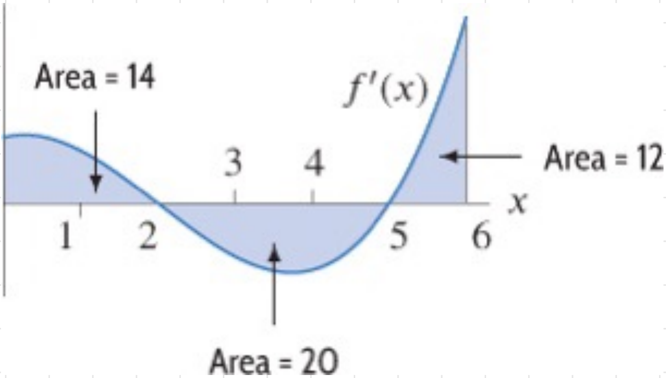
Review for Midterm on Thursday

Review for Final next Thursday

HW 12 Due Tonight (Late Submission allowed)

HW 13 Due April 27 ↖ covers Midterm 4 material.

HW 12 5.3 #6 Similar to it.



The fig shows $f'(x)$ of a function $f(x)$ and the value of some areas. If $f(0) = 10$, sketch $f(x)$.

Soln. Observations:

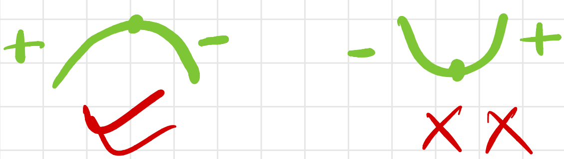
f is increasing on $(0, 2)$. as $f' > 0$

f is decreasing on $(2, 5)$ as $f' < 0$

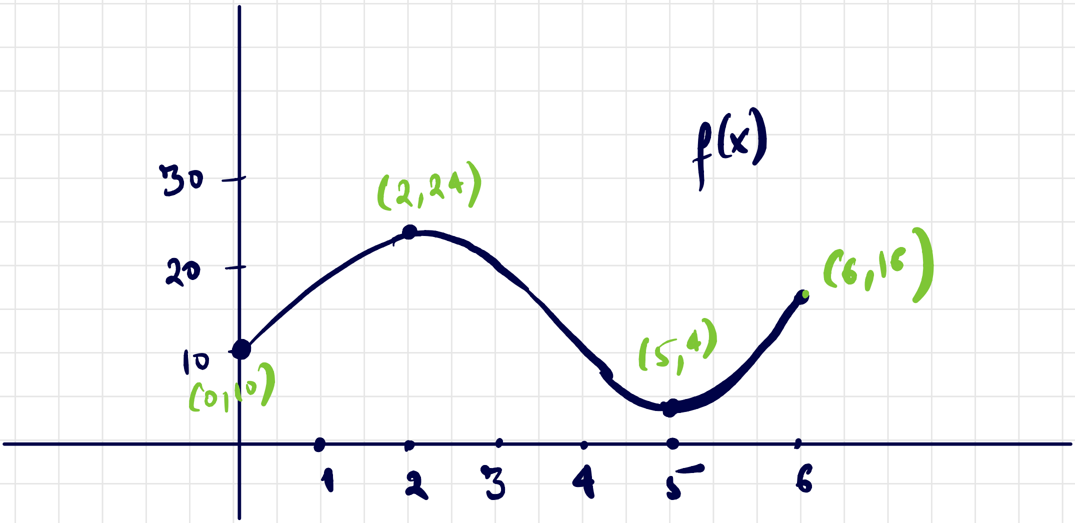
f is increasing on $(5, 6)$ as $f' > 0$.

$x = 2, 5$ are critical points as $f'(2) = 0$
$f'(5) = 0$

$x = 2$ is a local maximum.



$x = 5$ is a local minimum.



By FTC,

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

Since we know the value of $f(0)$, we take $a = 0$.

$$\int_0^b f'(x)\,dx = f(b) - f(0)$$

$$\Rightarrow \int_0^b f'(x)\,dx = f(b) - 10$$

$$\Rightarrow f(b) = \int_0^b f'(x)\,dx + 10 \qquad (*)$$

Take $b=2$

$$f(2) = \int_0^2 f'(x)\,dx + 10$$

From the fig.

$$= 14 + 10$$

$$= 24.$$

Take $b=5$ in $(*)$

$$f(5) = \int_0^5 f'(x)\,dx + 10$$

From the fig.

$$= (14 - 20) + 10$$

$$= -6 + 10$$

$$= 4$$

Take $b=6$ in $(*)$

$$f(6) = \int_0^6 f'(x)\,dx + 10$$

$$= (14 + 12 - 20) + 10$$

$$= 6 + 10$$

$$= 16.$$

§ 6. Antiderivatives and Applications

6.1. Analyzing Antiderivatives

$F(x)$. Say that derivative of $F(x)$ is $f(x)$, i.e.

$$F'(x) = f(x).$$

Then, $F(x)$ is called an antiderivative of $f(x)$.

An antiderivative of $f(x)$ is a function whose derivative is equal to $f(x)$.

Important because there could be several antiderivatives.

Ex.

$$F(x) = x^2$$

$$F'(x) = f(x) = \frac{d}{dx} x^2$$

$$= 2x$$

So, $f(x) = 2x$.

$2x$ is the derivative of x^2.

But, x^2 is an antiderivative of $2x$.

x^2+2 is an antiderivative of $2x$

Problem 1. Suppose $F'(t) = (1.8)^t$ and $F(0) = 2$. Find the value for $b = 0.1, 0.2$.

Solution: Want: $F(0.1)$, $F(0.2)$.

Given: $F'(t) = 1.8^t$

$F(0) = 2$.

By Fundamental Thm of Calculus,

$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

Since we know the value of $F(0)$ we take $a = 0$:

$$\int_0^b (1.8)^t\,dt = F(b) - F(0)$$

$$\Rightarrow \quad F(b) = \int_0^b (1.8)^t\,dt + 2 \quad (*)$$

So, $F(0.1) = \int_0^{0.1} (1.8)^t\,dt + 2$

↑ Use Calculator

$= \boxed{2.103}$

So, $F(0.2) = \int_0^{0.2} (1.8)^t\,dt + 2$

$= \boxed{2.212}$

6.2. Antiderivatives and the Indefinite Integral

$$\frac{d}{dx}(x^2) = 2x$$

Notice

$$\frac{d}{dx}(x^2+2) = \frac{d}{dx}x^2 + \cancelto{0}{\frac{d}{dx}2}$$

$$= 2x$$

$$\frac{d}{dx}(x^2-10) = 2x$$

$\frac{d}{dx}(x^2+c) = 2x$ for any constant c.

So, the antiderivatives of $2x$ are of the form

x^2+c where c is a constant.

GOAL: TO FIND ANTIDERIVATIVES OF FUNCTIONS. LOOKING AT THE ABOVE EXAMPLE WE SEE THAT

1. FIND JUST ONE ANTIDERIVATIVE.
2. ALL ANTIDERIVATIVES = SOME ANTIDERIVATIVE + Constant

Notation:

All the antiderivatives of $f(x)$ is denoted as

$$\int f(x)\,dx.$$

$\int f(x)\,dx$ is called the indefinite integral of $f(x)$.

By definition,

$$\int f(x)\,dx = F(x) + C$$

where $F(x)$ is an antiderivative of $f(x)$.

From the first example,

$$\int 2x\,dx = x^2 + C$$

Formulas:

- Notice $\frac{d}{dx} x^2 = 2x$

$$\Rightarrow \frac{d}{dx}\left(\frac{x^2}{2}\right) = x$$

An antiderivative of x is $\frac{x^2}{2}$

$$\frac{d}{dx} x^3 = 3x^2$$

$$\Rightarrow \frac{d}{dx}\left(\frac{x^3}{3}\right) = x^2$$

An antiderivative of x^2 is $\frac{x^3}{3}$.

In general, an antiderivative of x^n is $\frac{x^{n+1}}{n+1}$.

So,

1. $$\boxed{\int x^n \, dx = \frac{x^{n+1}}{n+1} + C}$$

Power Rule for antiderivatives.

$n \neq -1$

- $\int k\,dx = ?$ $\quad$ k is constant.

Q. What is a function whose derivative equals k?

A. $\frac{d}{dx}(kx) = k\frac{d}{dx}(x)$

$= k$

So, kx is an antiderivative of k

Thus,

2. $$\boxed{\int k\,dx = kx + C}$$

Antiderivative of constant

- If you plug $n=-1$ into the power Rule,

$$\int x^{-1}\,dx = \frac{x^{-1+1}}{-1+1} + C$$

So, $$\int \frac{1}{x}\,dx = \frac{x^0}{0} + C$$

This is not well-defined.

The power rule does not work when $n=-1$.

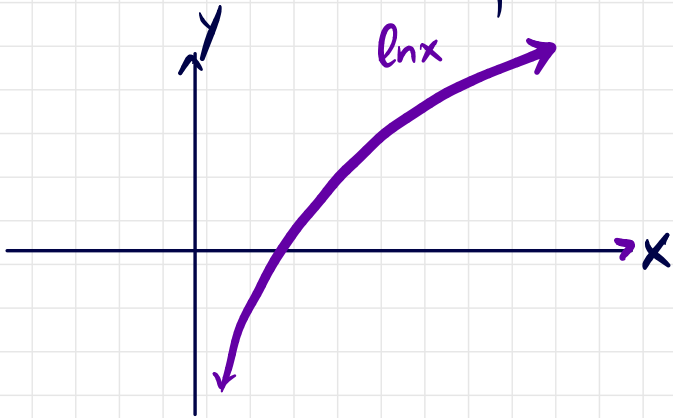
Want: $$\int \frac{1}{x}\,dx = ?$$

Recall, $$\frac{d}{dx}\ln x = \frac{1}{x}$$

Hence, $\ln x$ is an antiderivative of $\frac{1}{x}$.

So, 1. $$\int \frac{1}{x}\,dx = \ln x + C$$

for $x > 0$



If $x < 0$, $\ln x$ is not defined.

Try $\ln(-x)$.

$$\frac{d}{dx} \ln(-x) = \frac{1}{x}$$

So, 2. $\int \frac{1}{x}\, dx = \ln(-x) + C$, for $x < 0$.

Combining both 1 and 2 we get

3. $$\boxed{\int \frac{1}{x}\, dx = \ln|x| + C}$$

Antiderivative of reciprocal.

x positive. $|x| = x$ so you get formula 1.

x negative. $|x| = -x$ so yo uget $\ln(-x)$ on the R.H.S, which is formula 2.

$$\left[\begin{aligned} |-3| &= -(-3) \\ &= 3 \end{aligned} \right]$$

- $$\frac{d}{dx} e^x = e^x$$

So,

4₁. $$\boxed{\int e^x \, dx = e^x + C}$$

$$\frac{d}{dx} e^{kx} = k e^{kx}$$

Exponential Rules.

$$\Rightarrow \frac{d}{dx}\left(\frac{e^{kx}}{k}\right) = e^{kx}$$

So, $\frac{e^{kx}}{k}$ is an antiderivative of e^{kx}.

So,

4₂: $$\boxed{\int e^{kx} \, dx = \frac{e^{kx}}{k} + C}$$

Properties:

1. $$\int f(x) \pm g(x) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

2. $$\int c \, f(x) \, dx = c \int f(x) \, dx$$

Problem 1.

Find $\int 3x + x^2 \, dx$.

Soln.

$$\int 3x + x^2 \, dx$$

$$= \int 3x \, dx + \int x^2 dx \quad \text{(Property 1)}$$

$$= 3 \int x^1 dx + \int x^2 dx \quad \text{(Property 2)}$$

$$= 3\left(\frac{x^{1+1}}{1+1} + C_1\right) + \frac{x^{2+1}}{2+1} + C_2 \quad \text{(Power Rule)}$$

$$= \frac{3x^2}{2} + 3C_1 + \frac{x^3}{3} + C_2$$

$$= \frac{3x^2}{2} + \frac{x^3}{3} + \underbrace{3C_1 + C_2}$$

$$= \boxed{\frac{3x^2}{2} + \frac{x^3}{3} + C}$$

where C is a constant