Midterm 4 April 27

HW 12 April 20

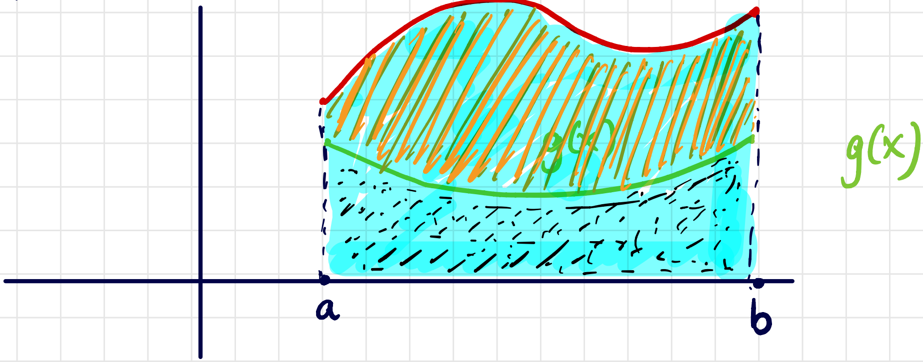
<u>Calculator Instructions:</u>

$$\int_{-4}^{5} e^{-x^2}\,dx = ?$$

1. $\boxed{Y}= e^{-x^2}$
2. Press $\boxed{\text{GRAPH}}$
3. Press $\boxed{\text{2ND}}$ + $\boxed{\text{TRACE}}$
4. Press 7 $\int f(x)\,dx$
5. Enter -4 (lower limit)
6. Enter 5 (upper limit
7. $\int f(x)\,dx = \boxed{1.7724}$

Area Between Two Curves

If $g(x) \leq f(x)$



We want to find the area between $f(x)$ and $g(x)$ from a to b.

Area between $f(x)$ and $g(x)$ = Area under $f(x)$ − Area under $g(x)$

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

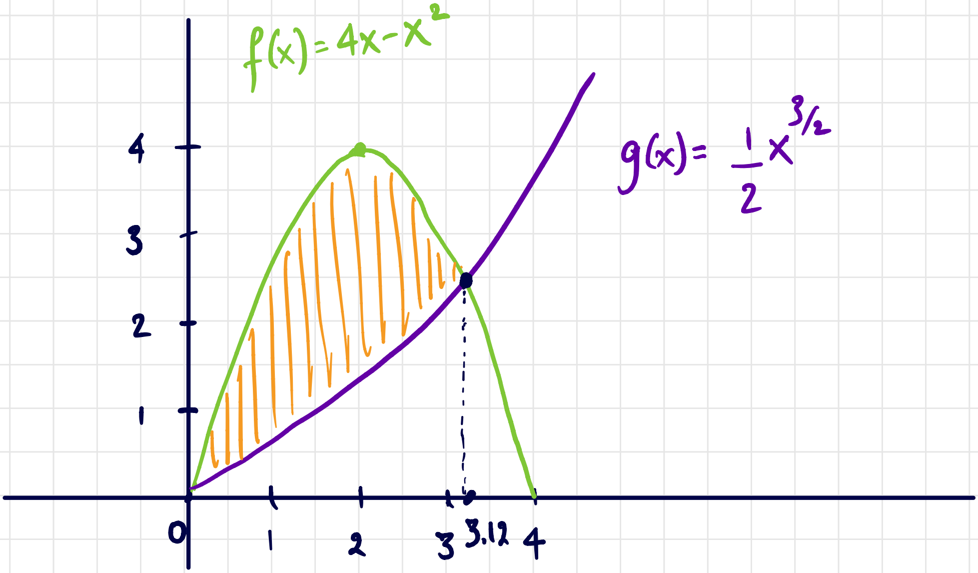
$$= \int_a^b f(x) - g(x)\,dx$$

$$\int_a^b f+g\,dx$$

$$= \int_a^b f\,dx + \int_a^b g\,dx$$

Problem 1 Let $f(x) = 4x - x^2$, $g(x) = \frac{1}{2}x^{3/2}$.

The graphs for $x \geq 0$ are shown. Find the area enclosed by the two curves.



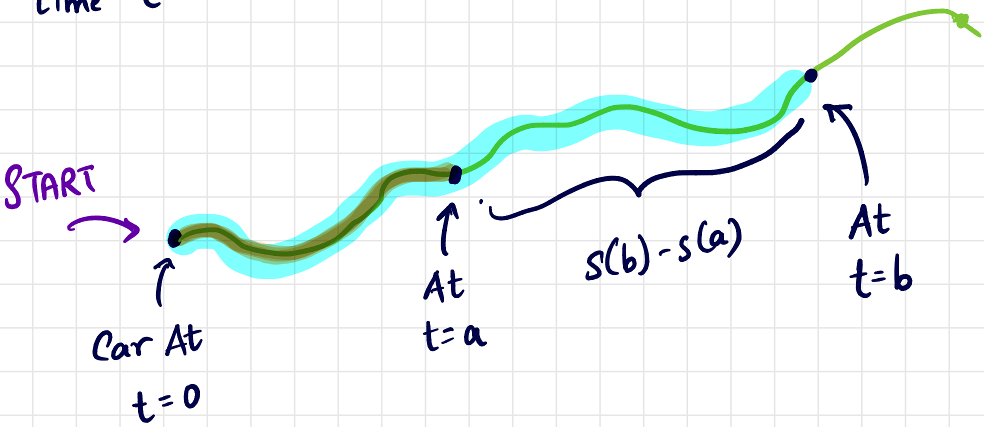
Solution. Want Area of

$$\text{Area enclosed by two curves} = \int_0^{3.12} 4x - x^2 - \frac{1}{2}x^{3/2}\,dx$$

$$= \boxed{5.906}$$

§5.5 Fundamental Theorem of Calculus

time t



Let $s(t)$ denote the distance travelled by the car starting from the starting point.

$$s(0) = 0$$

$s(a)$ = distance travelled from 0 to a.

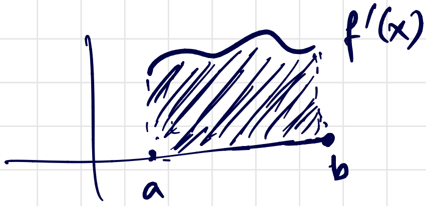
We Know that $\quad s'(t) = v(t) \quad$ (velocity)

$$\int_a^b v(t)\,dt = \text{Distance travelled between } t=a \text{ and } t=b$$

$$\Rightarrow \boxed{\int_a^b s'(t)\,dt = s(b) - s(a)}$$

[Newton/Leibniz] If $f(x)$ is continuous and $f'(x)$ is continuous on $a \leq x \leq b$, then

$$\boxed{\int_a^b f'(x)\,dx = f(b) - f(a)}$$



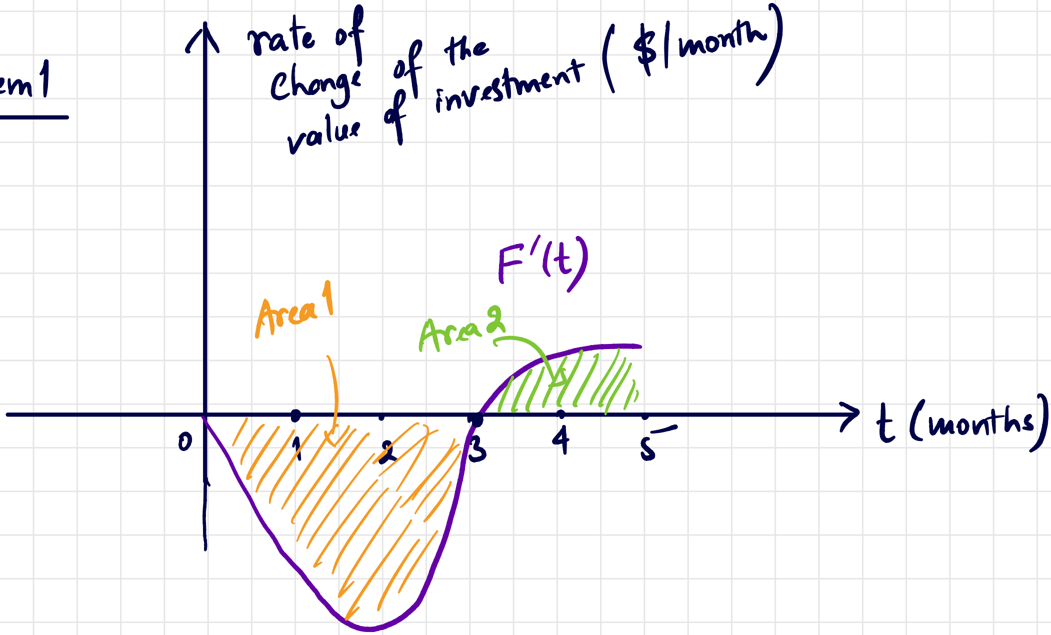
Area = $f(b) - f(a)$

Intuition: L.H.S. = Integrating the rate of change function.

We know that if we integrate the rate of change function we get the total change from a to b.

But total change from a to b $= f(b) - f(a)$

Problem 1



$F'(t)$, the rate of change of an investment $F(t)$ over 5-month period.

a) When is the value of the investment increasing and when is it decreasing?

Soln. Recall.

$F(t)$ is increasing on interval $\Leftrightarrow$ $F'(t) > 0$ on that interval

$F(t)$ is decreasing on interval $\Leftrightarrow$ $F'(t) < 0$ on that interval.

So, $F(t)$ is increasing on $(3, 5)$

$F(t)$ is decreasing on $(0, 3)$

b) Does the investment increase or decrease in value during the 5 months?

Soln. Want: Total change is positive or negative?

Total change of $F(t)$ from 0 to 5 $= \displaystyle\int_0^5 F'(t)\,dt$

By FTC $= F(5) - F(0)$ $(*)$

(we don't know these values)

But, $\displaystyle\int_0^5 F'(t)\,dt = -\text{Area}\,1 + \text{Area}\,2$

Since Area 1 is bigger,

$\displaystyle\int_0^5 F'(t)\,dt$ is negative.

By $(*)$, $F(5) - F(0)$ is negative.

$\Rightarrow F(5) - F(0) < 0$

$\Rightarrow F(5) < F(0)$

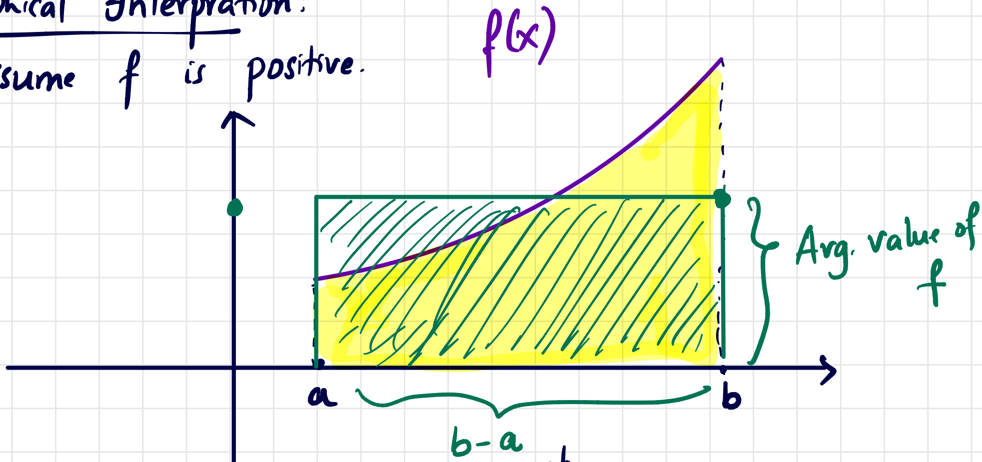
The investment decreases in value.

§ 5.6. Average Value

Average value of f on the interval from a to b $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Graphical Interpration:

Assume f is positive.



$$\text{Avg. value of } f = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\Rightarrow (\text{Avg. value of } f)\,(b-a) = \int_a^b f(x)\,dx \quad (*)$$

R.H.S. = Area under $f(x)$.

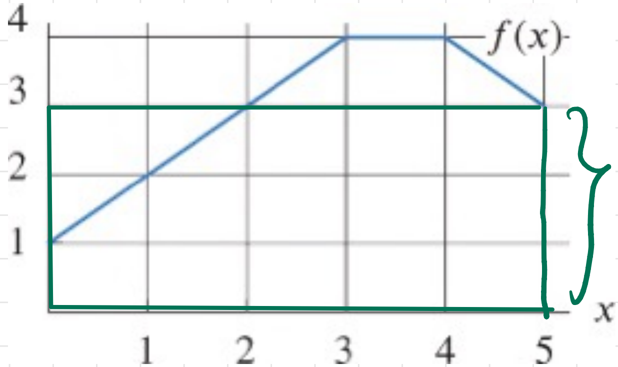
L.H.S. = $(\underbrace{\text{Avg. value of } f}_{\text{some number}})\ (b-a)$

= Area of rectangle whose base is $b-a$ and height is Avg. value. 

 =

Problem 1

a) $f(x)$ is graphed below. Evaluate $\int_0^5 f(x)\,dx$.



Avg. value $= 3$

Solution.

$$\int_0^5 f(x)\,dx = \text{Area under the graph}$$

$$= 13 \text{ squares} + 4 \text{ half-squares}$$

$$= 15 \text{ squares}$$

But each square has area $1 \times 1 = 1$ (unit area).

Thus,

$$\int_0^5 f(x)\,dx = \boxed{15}$$

b) Find the avg. value of $f(x)$ on the interval $x = 0$ to $x = 5$ and check the answer graphically.

Soln.

Avg. value of f from 0 to 5 $= \dfrac{1}{5-0} \displaystyle\int_0^5 f(x)\,dx$

$$= \frac{1}{5} \cdot 15 = \boxed{3}$$