Midterm 4. April 27., Tuesday

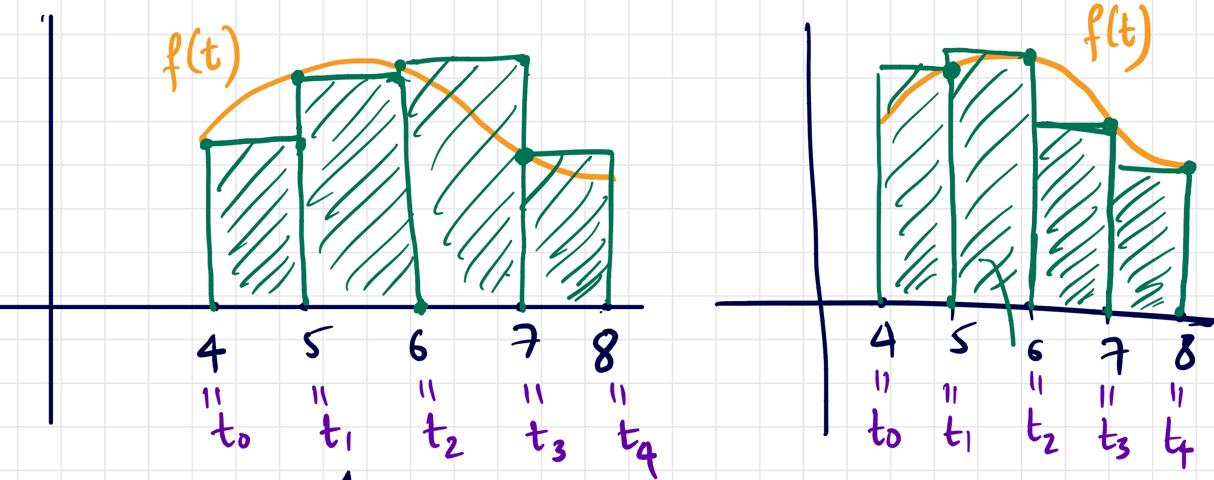
Final. Monday, May 3. 8:00- 10:30

HW 11 Due Tonight

You can submit anytime.

Office Hrs. TR 10:45 - 12:30

I will post HW 12.



$n = 4$

$$\Delta t = \frac{8-4}{4} = \frac{4}{4} = 1$$

Left Hand sum $= f(4) \cdot \Delta t + f(5)\Delta t + f(6)\Delta t + f(7)\Delta t$

$= f(4) + f(5) + f(6) + f(7).$

Right Hand sum $= f(5)\Delta t + f(6)\Delta t + f(7)\Delta t + f(8)\Delta t$

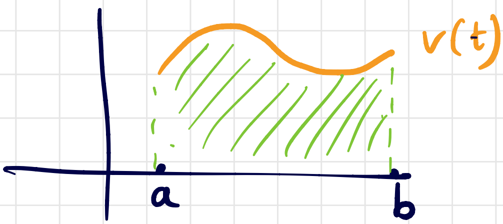
$= f(5) + f(6) + f(7) + f(8)$

5.2. The Definite Integral

We concluded that if f is a rate of change of some quantity, then the left hand sum and the right hand sum approximate the total change in the quantity.

Ex. distance function

rate of change of distance = velocity = $v(t)$.



Total change in distance

= Distance travelled in the time interval $[a, b]$

$\approx$ left hand sum, right hand sum.

approx.

$\approx$ Area under the velocity graph.

approx.

The approximation is improved by the increasing the value of n. To find the total change exactly we take larger and larger values of n. and look at the value approached by the left hand and right hand sums.

$n = 10$, $n = 10{,}000$, $n = 10{,}000{,}000$ etc. ...

This is called taking the limit as n goes to infinity.

Def. Suppose f is continuous for $a \leq t \leq b$.
The definite integral of f from a to b denoted as $\int_a^b f(t)\,dt$ is the limit of the left hand sum or the right hand sums with n subdivisions of $[a,b]$ as n gets arbitrarily large. (Limit of L.H.S. = R.H.S when integral exists)

$$\int_a^b f(t)\,dt = \lim_{n\to\infty}(\text{left hand sum}) = \lim_{n\to\infty}(\text{right hand sum})$$

$$= \lim_{n\to\infty} \sum_{i=0}^{n-1} f(t_i)\Delta t = \lim_{n\to\infty} \sum_{i=0}^{n} f(t_i)\Delta t$$

These sums are called Riemann sums.

f is called integrand.

a, b are called limits of integration.

$\int$ introduced by Leibniz. Inspired by $\sum$

$$\int_a^b f(t)\,dt \xleftrightarrow{\text{analagos}} \sum_{i=0}^{n-1} f(t_i)\,\Delta t$$

$\parallel$ $dt = \Delta t$ when Δt is infinitely small

infinitesimal.

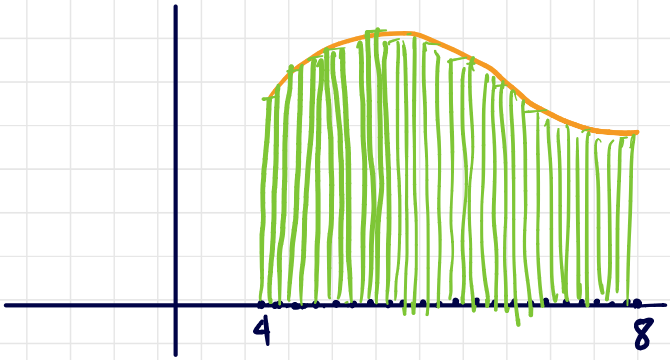
Infinite sum $\parallel$

$$= f(t_0)\Delta t + f(t_1)\Delta t + f(t_2)\Delta t \cdots + f(t_{n-1})\Delta t$$

dt is not a number

doesn't make sense by itself.

The definite integral is the area under the graph from a to b (with some minor exceptions).



First example

$n =$ Very large. (say 1000)

Δt = very small

$n = 4$ vs. $n = 1000$

Left hand sum

Right hand sum

What did we gain by increasing the value of n?

We got closer to the total change or to the area under the graph.

$n = 1000$, $n = 10,0000$, $n = 10,000,000$ — 80 m.

Problem 1

Compute $\int_0^1 e^{-t^2}\,dt$ and represent this integral as an area.

Soln. $\int_0^1 e^{-t^2}\,dt = \lim_{n\to\infty}$ left hand sum or

$\lim_{n\to\infty}$ right hand sum

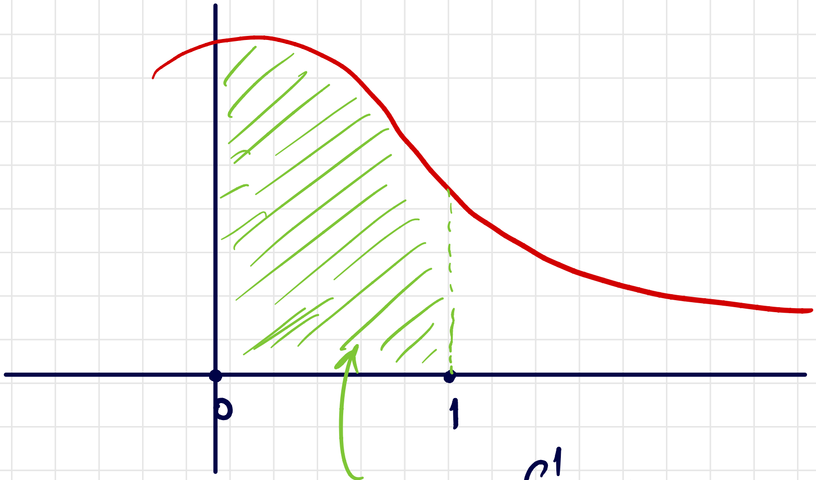
$$\sum_{i=0}^{10,000} f(t_i)\,\Delta t = ?$$

1. 2ND TRACE
2. $\int f(x)\,dx$ (No 7 in TI-84)
3. $X = 0$
4. $X = 1$
5. e^{-X^2}

$$\int_0^1 e^{-t^2}\,dt = \boxed{0.747}$$

WINDOW

$\Delta X = 0.2$



Area $= \int_0^1 e^{-t^2}\,dt$

Problem 2 Estimate $\int_{20}^{30} f(t)\,dt$

t	20	22	24	26	28	30
$f(t)$	5	7	11	18	29	45

Exercise

Left Hand Sum

Right Hand Sum

$$\text{Estimate} = \frac{\text{Left hand sum} + \text{right hand sum}}{2}$$

$n = 5$

$\Delta t = 2$

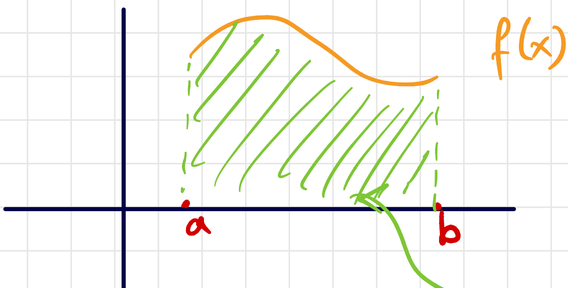
$$\Delta t = \frac{b-a}{n} = \frac{30-20}{5} = \frac{10}{5} = 2$$

5.3. The Definite Integral as Area

I said earlier that $\int_a^b f(x)\,dx$ is the area under the graph of f from a to b.

But this is not quite true. We have to some sign changes when the graph is negative.

Positive statement is true:

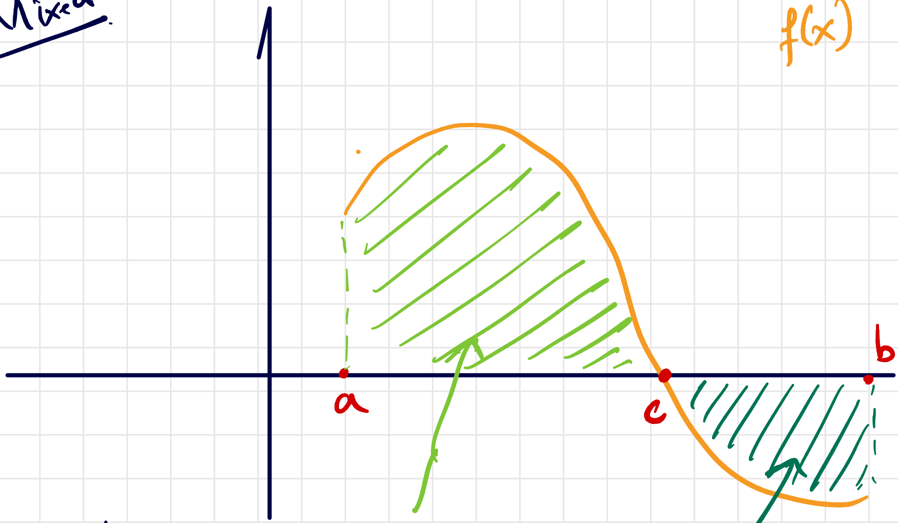


$$\int_a^b f(x)\,dx = \text{Area}$$

Negative

$$\int_a^b f(x)\,dx = -\text{Area}$$

<u>Mixed</u>

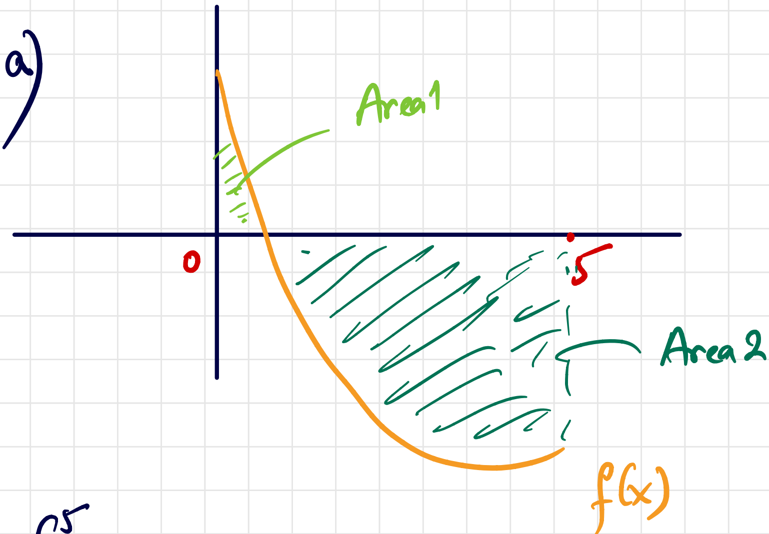


$$\int_a^b f(x)\,dx = \text{Area 1} - \text{Area 2}$$

$$= \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Problem 1 For each of the following graphs, decide whether $\int_0^5 f(x)\,dx$ is positive, negative or approximately zero.

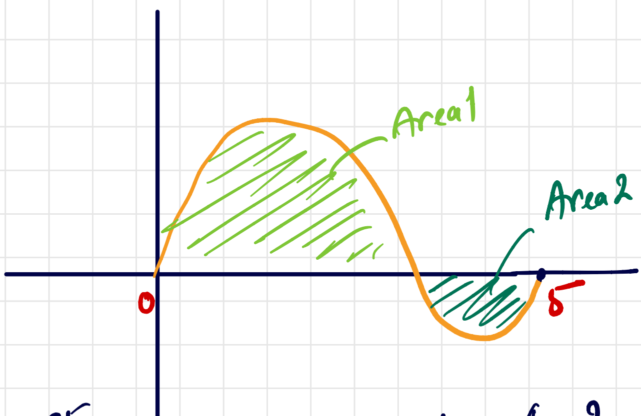
a)



$\int_0^5 f(x)\,dx$ is <u>negative</u>

$= \text{Area1} - \text{Area2}$. Area2 dominates Area1.

b)

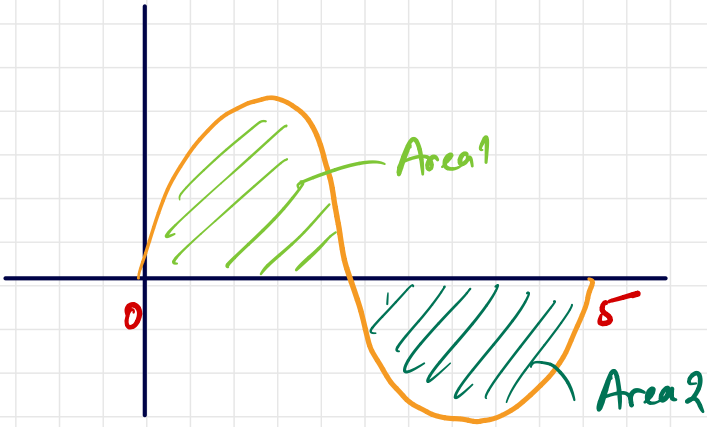


Area1 is bigger

$\int_0^5 f(x)\,dx$ ~~is~~ $= \text{Area1} - \text{Area2}$

$\Rightarrow$ positive.

c.



$\int_0^5 f(x)\,dx$ is approx. zero.

Area 1 is almost the same as Area 2.

Since $\int_0^5 f(x)\,dx = \text{Area 1} - \text{Area 2}$

≈ 0