Modeling Content Spreading On Networks Through Network Science And Graph Neural Networks

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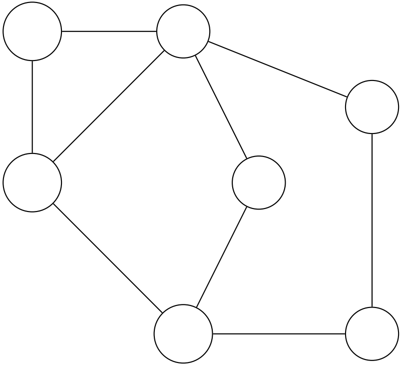
Kansas State University

Joint Mathematics Meetings
January 10, 2025

Model of Content Spreading

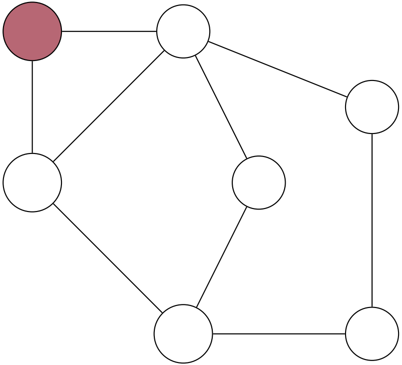


H.Z. Brooks, M.Porter (2024)

An "Opinion Reproduction Number" for Infodemics in a Bounded-Confidence Content-Spreading Process on Networks. arxiv:2403.01066

- Undirected network

Model of Content Spreading



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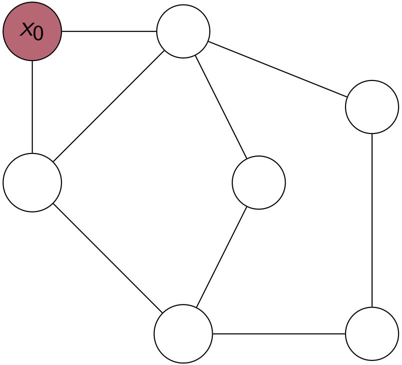

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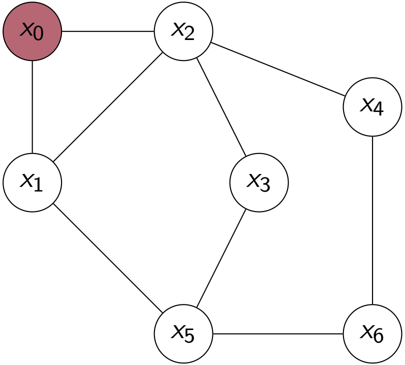

- Undirected network
- Choose source node 0 uniformly at random
- Content $x_0 \in [0, 1]$ starts at node 0
- Initialize other nodes with opinion states $x_i \in [0, 1]$ chosen from some distribution

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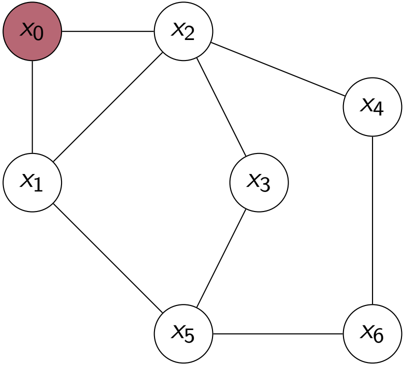

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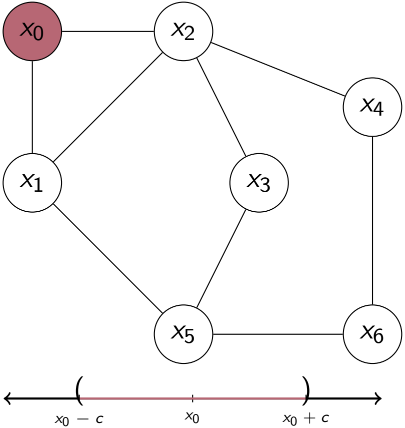

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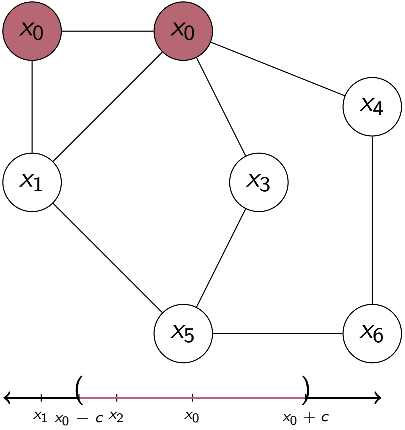

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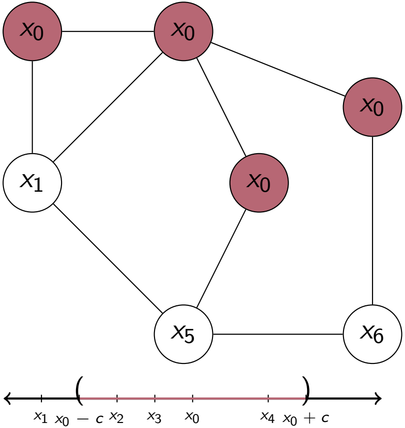

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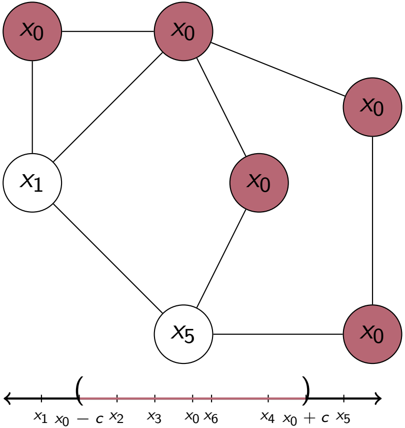

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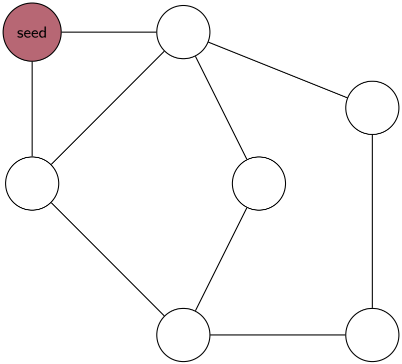
Site percolation is a special case



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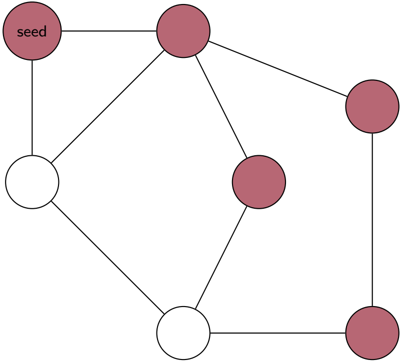

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- Every node is open with probability p and closed with probability $1 - p$.

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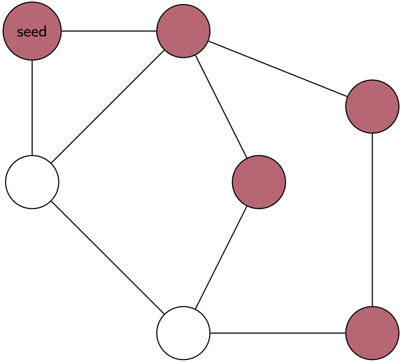

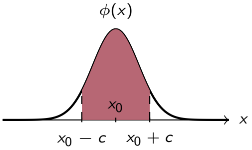

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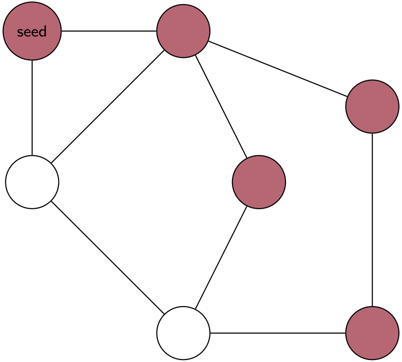

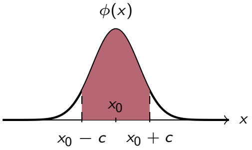

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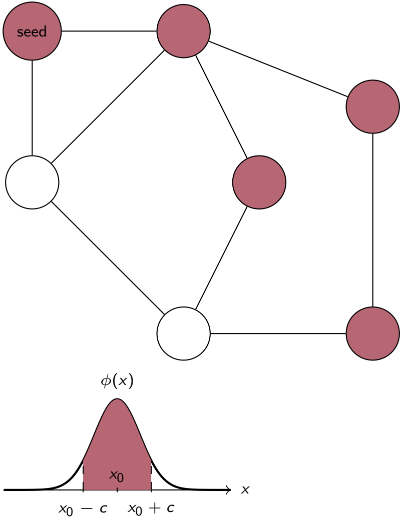

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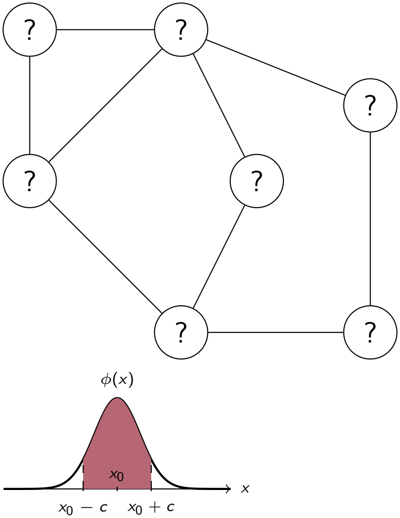
- p is called occupation probability.
- Influenced nodes are nodes that are contained in the "open" cluster containing the seed.

Two Questions



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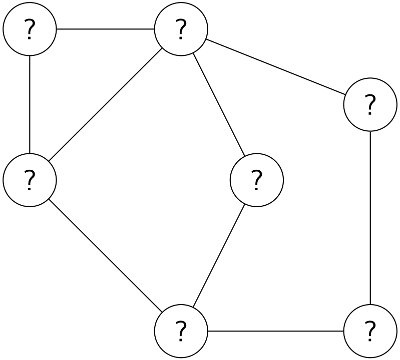

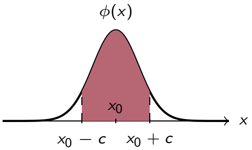

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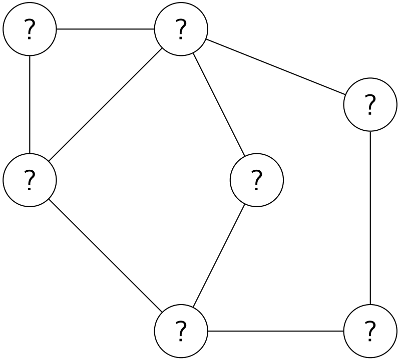

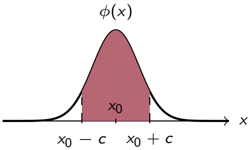

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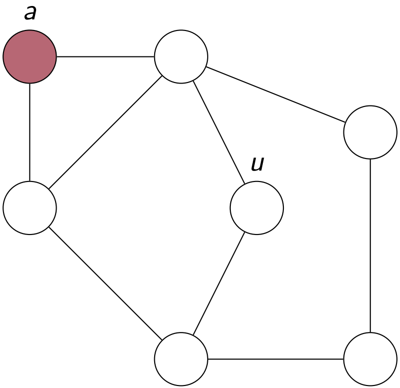
- **Influence Maximization:** What is the best seed to start the diffusion to maximize influence?
  $\mathbb{E}$[size of cluster containing seed]
- **Influence Computation:** Given a seed what is the probability that a node is influenced?
  $\mathbb{P}_{\text{seed}}(v \text{ is influenced})$ for $v \in V$.
  (Note: This does not refer to occupation probability)

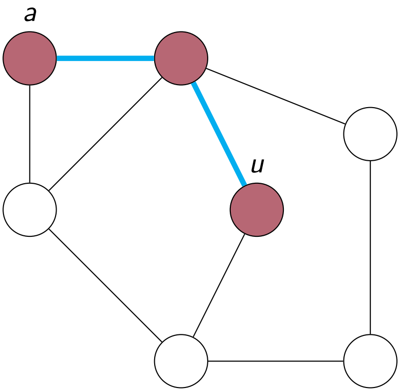
Example



- a is the seed node. What is $\mathbb{P}_a(u \text{ influenced})$?
- Assume we know the "openness/occupation" probability p.

Example

Consider this path



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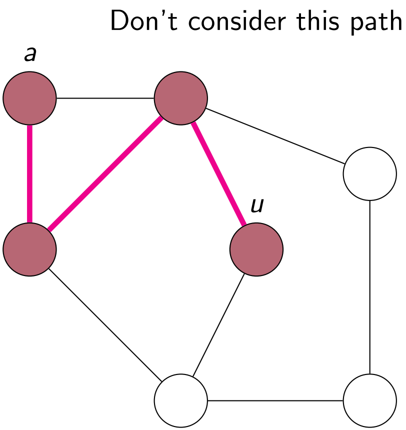

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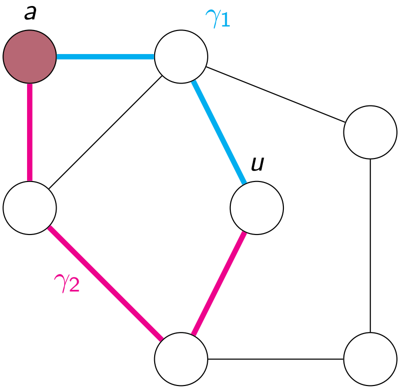
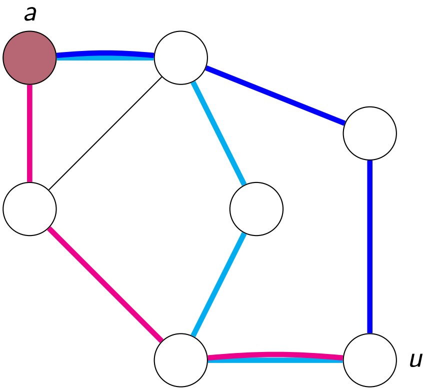


- a is the seed node. What is $\mathbb{P}_a(u \text{ influenced})$?
- Assume we know the "openness/occupation" probability p.
- Two paths γ_1 and γ_2. So

$$\begin{aligned}\mathbb{P}_a(u) &= \mathbb{P}(\gamma_1 \cup \gamma_2) \\ &= \mathbb{P}(\gamma_1) + \mathbb{P}(\gamma_2) - \mathbb{P}(\gamma_1 \cap \gamma_2) \\ &= p^2 + p^3 - p^5\end{aligned}$$

General Case



- Difficult: have to consider all chord-less paths and their intersections.

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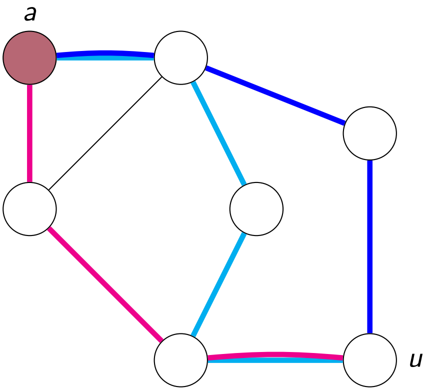

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- Monstrous calculations (Bonferroni type):

$$\mathbb{P}(\gamma_1 \cup \gamma_2 \cup \gamma_3) = \mathbb{P}(\gamma_1) + \mathbb{P}(\gamma_2) + \mathbb{P}(\gamma_3) - \mathbb{P}(\gamma_1 \cap \gamma_2) - \mathbb{P}(\gamma_1 \cap \gamma_3) - \mathbb{P}(\gamma_2 \cap \gamma_3) + \mathbb{P}(\gamma_1 \cap \gamma_2 \cap \gamma_3)$$

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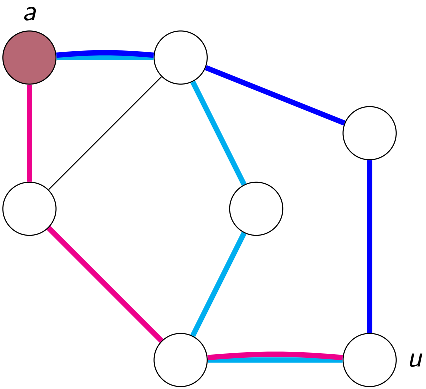

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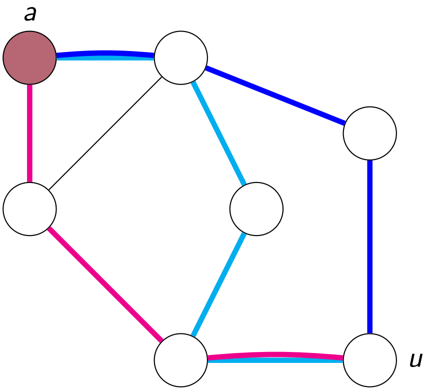

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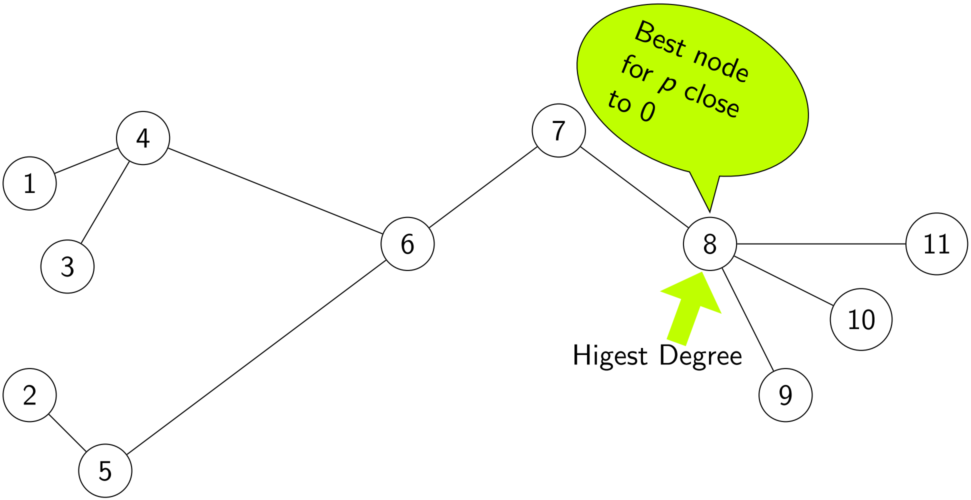
- Paths might have intersections which implies correlations between events.
- Influence maximization is NP-hard, and influence computation is #P-hard.

Trees are more tractable



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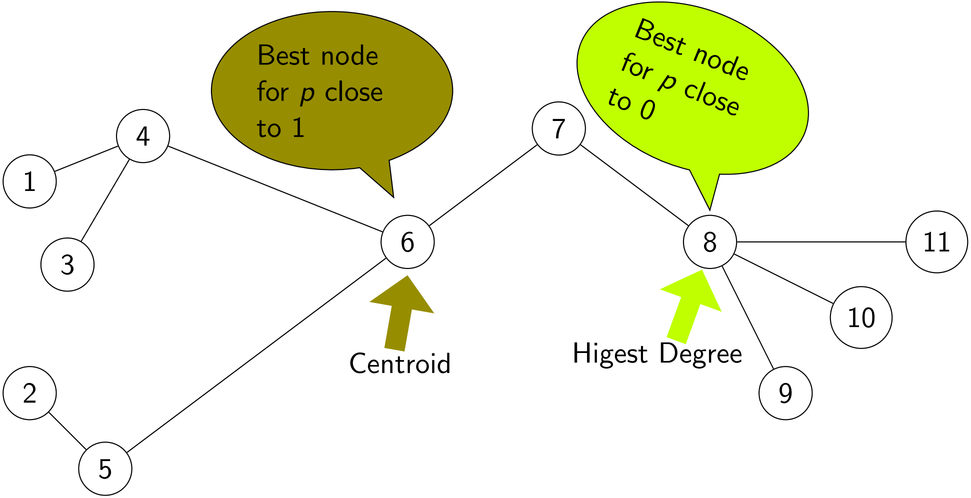
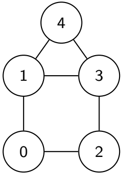


Towards Deep Learning: Node2Vec for input embeddings

Goal

Train a model to learn influence probabilities.



Graph

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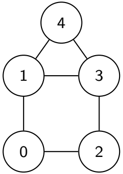

Graph

Random Walks

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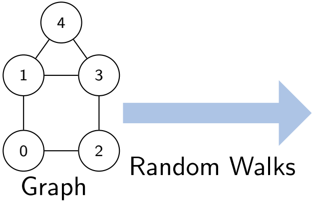

Sentences:
$[0, 2, 3, 4, 1]$,
$[1, 0, 2, 0]$,...

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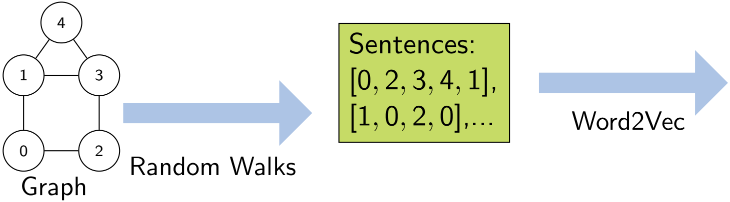

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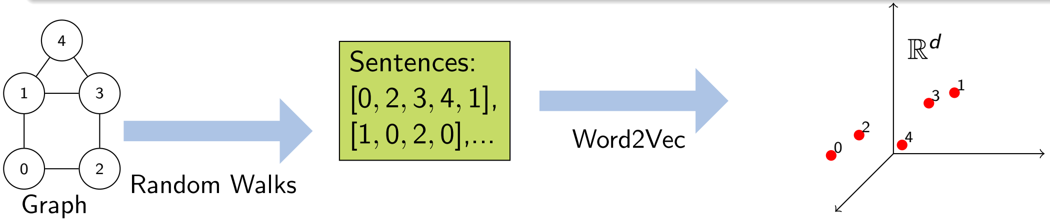

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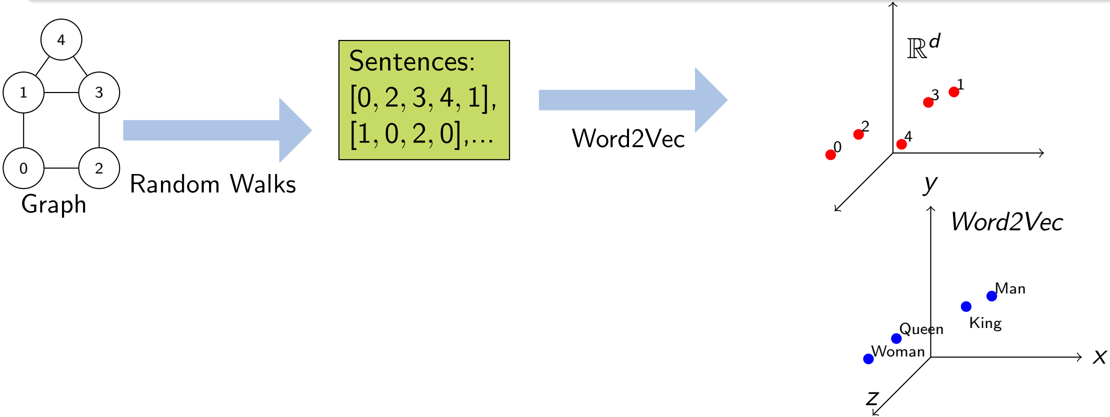
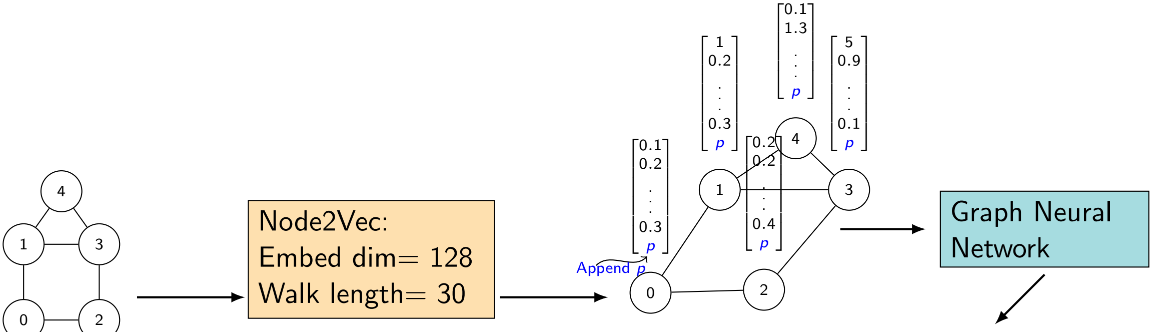
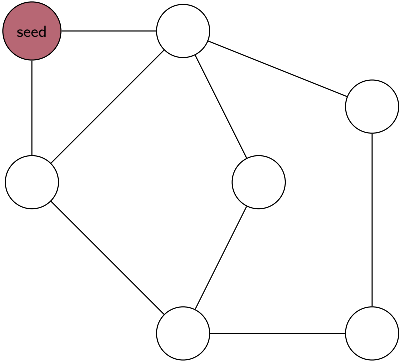


Model Architecture



$$
\begin{array}{c}
\mathbb{P}_0 \\ \mathbb{P}_1 \\ \mathbb{P}_2 \\ \mathbb{P}_3 \\ \mathbb{P}_4
\end{array}
\begin{bmatrix}
1 & 0.2 & 0.2 & 0.08 & 0.04 \\
0.2 & 1 & 0.04 & 0.2 & 0.2 \\
0.2 & 0.04 & 1 & 0.2 & 0.04 \\
0.8 & 0.2 & 0.2 & 1 & 0.04 \\
0.04 & 0.1 & 0.04 & 0.04 & 1
\end{bmatrix}
$$

Synthetic Dataset



- Monte Carlo Simulations

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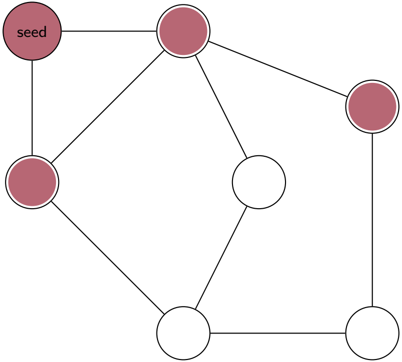

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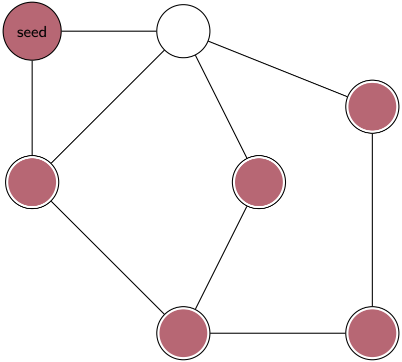

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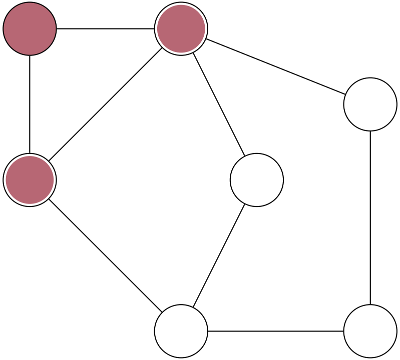

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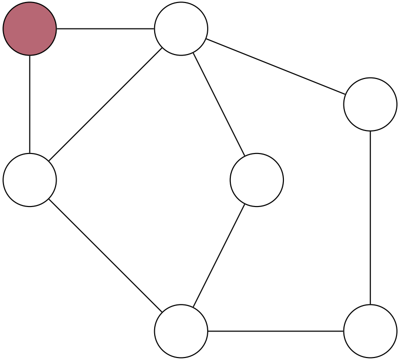

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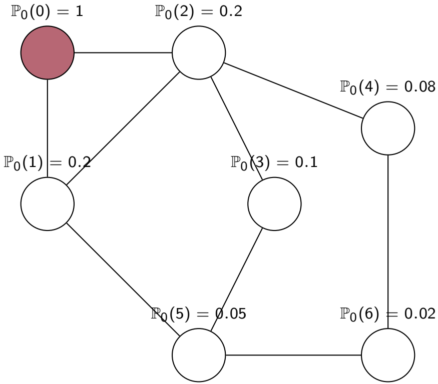

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  Get first row of matrix: For $p = 0.2$, $[\mathbb{P}_0(0), \mathbb{P}_0(1), ..., \mathbb{P}_0(6)]$

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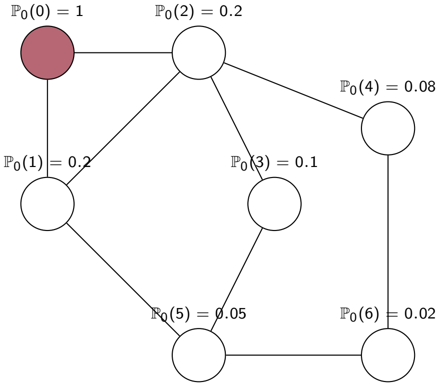

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- 1000 random graphs of sizes 8-50 (random geometric, erdos-renyi, etc.).

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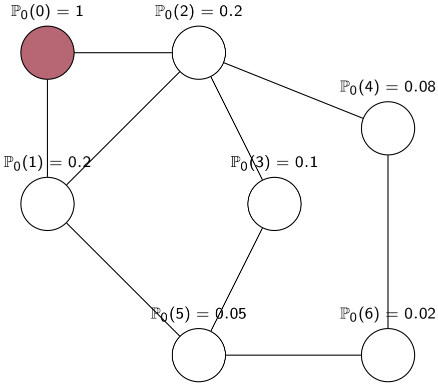

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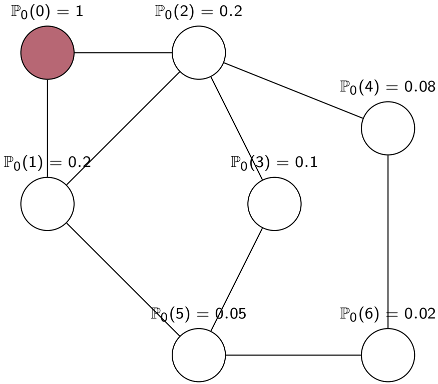

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- 90-10 train-test split.

GNN Parameters and Results

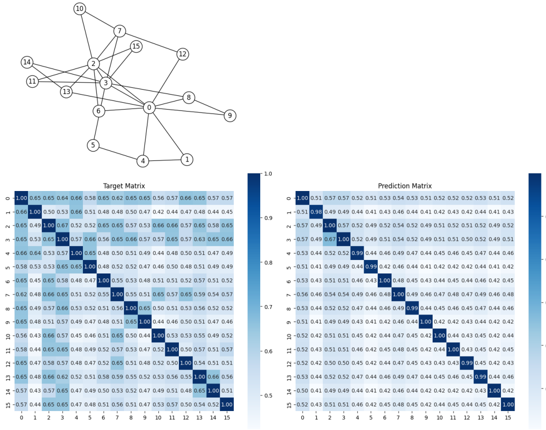

Figure: Sample graph with target and predicion matrices for $p = 0.65$

- Loss Function: Mean Squared Error (MSE)
- Graph neural network parameters:
  Number of layers: 3 (dropout 0.1)
  Hidden Dimension: 256
  Batch: 4
- Training time $\sim$15 minutes (GPU). Early stopping after 23 epochs
- Train Loss: MSE 0.0063
  Test Loss: MSE 0.020

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- Apply the methods from this work to other models in opinion dynamics and epidemic modeling.

Thank You!